

Chapter 3

Radian Measure and Circular Functions

Section 3.1: Radian Measure

- 1.
- 2.
- 3.
4. -1
5. -3
6. -2
7. $\frac{\pi}{3}$
8. $\frac{\pi}{6}$
9. $\frac{\pi}{2}$
10. $\frac{2\pi}{3}$
11. $\frac{5\pi}{6}$
12. $\frac{3\pi}{2}$
13. $-\frac{5\pi}{3}$
14. $-\frac{7\pi}{4}$
15. $\frac{5\pi}{2}$
16. $\frac{8\pi}{3}$
17. 10π
18. -20π
19. Multiply the degree measure by $\frac{\pi}{180}$ radian and reduce. Your answer will be in radians. Leave the answer as a multiple of π , unless otherwise directed.
20. Multiply the radian measure by $\frac{180^\circ}{\pi}$ and reduce. Your answer will be in degrees.
- 21.–24. Answers will vary.
25. 60°
26. 480°
27. 315°
28. 120°
29. 330°
30. 675°
31. -30°
32. -288°
33. 126°
34. 132°
35. -48°
36. -63°
37. 153°
38. 66°
39. -900°
40. 2700°
41. .68
42. 1.29
43. .742
44. 4.623
45. 2.43
46. 3.05
47. 1.122
48. 1.484
49. .9847
50. 2.140
51. .832391
52. .401675

53. $114^\circ 35'$

54. $286^\circ 29'$

55. $99^\circ 42'$

56. $175^\circ 20'$

57. $19^\circ 35'$

58. $564^\circ 14'$

59. $-287^\circ 06'$

60. $-198^\circ 55'$

61. Without the degree symbol on the 30, it is assumed that 30 is measured in radians. Thus, the approximate value of $\sin 30$ is $-.98803$, not $\frac{1}{2}$.

62. Answers will vary.

63. $\frac{\sqrt{3}}{2}$

64. $\frac{\sqrt{3}}{2}$

65. 1

66. $\frac{\sqrt{3}}{3}$

67. $\frac{2\sqrt{3}}{3}$

68. $\sqrt{2}$

69. 1

70. 1

71. $-\sqrt{3}$

72. $-\frac{\sqrt{3}}{3}$

73. $\frac{1}{2}$

74. $-\frac{\sqrt{3}}{3}$

75. -1

76. -1

77. $-\frac{\sqrt{3}}{2}$

78. $-\frac{\sqrt{3}}{3}$

79. $\frac{1}{2}$

80. $\frac{\sqrt{3}}{2}$

81. $\sqrt{3}$

82. $-\frac{2\sqrt{3}}{3}$

83. Begin the calculation with the blank next to 30° , and then proceed counterclockwise from there.

$$\frac{\pi}{6}; 45^\circ; \frac{\pi}{3}; 120^\circ; 135^\circ; \frac{5\pi}{6}; \pi; \frac{7\pi}{6}$$

$$\frac{5\pi}{4}; 240^\circ; 300^\circ; \frac{7\pi}{4}; \frac{11\pi}{6}$$

84. (a) $\frac{\pi}{200}$

(b) $3.15^\circ; .055$ radian

85. (a) 4π

(b) $\frac{2\pi}{3}$

86. (a) 16π

(b) 60π

87. (a) 5π

(b) $\frac{8\pi}{3}$

88. 24π

Section 3.2: Applications of Radian Measure

Answers will vary. The longitude at Greenwich is 0° .

Exercises

1. 2π

2. 4π

3. 20π

4. 8

5. 6

6. 8

7. 1
 8. 1.5
 9. 2
 10. Answers will vary.
 11. 25.8 cm
 12. 3.08 cm
 13. 3.61 ft
 14. 11.9 mi
 15. 5.05 m.
 16. 169 cm.
 17. 55.3 in.
 18. 71.4 ft
 19. The length of the arc is doubled.

20. $s = \frac{\pi r \theta}{180}$

For Exercises 21–26, note that since 6400 has two significant digits and the angles are given to the nearest degree, we can have only two significant digits in the answers.

21. 3500 km
 22. 1500 km
 23. 5900 km
 24. 8800 km
 25. 44° N
 26. 43° N
 27. 156°
 28. 213°
 29. 38.5°
 30. 82.3°
 31. 18.7 cm.
 32. 29.2 in.
 33. (a) 11.6 in.
 (b) $37^\circ 5'$.
 34. 12.7 cm.
 35. 146 in.
 36. (a) 39,616 rotations
 (b) 62.9 mph; Yes
37. .20 km
 38. 850 ft
 39. 6π
 40. 16π
 41. 72π
 42. 75π
 43. 60°
 44. 240°
 45. 1.5
 46. 1

In Exercises 47–54, we will be rounding to the nearest tenth.

47. 1116.1 m²
 48. 3744.8 km²
 49. 706.9 ft²
 50. 10,602.9 yd²
 51. 114.0 cm²
 52. 365.3 m²
 53. 1885.0 mi²
 54. 19,085.2 km²
 55. 3.6
 56. 16 m
 57. (a) $13\frac{1}{3}^\circ$; $\frac{2\pi}{27}$
 (b) 478 ft.
 (c) 17.7 ft.
 (d) ≈ 672 ft²
 58. 75.4 in.².
 59. (a) 140 ft
 (b) 102 ft
 (c) 622 ft²
 60. 550 m; 1800 m
 61. 1900 yd²
 62. 1.15 mi

63. radius: 3950 mi.; circumference: 24,800 mi. (c) 0
64. approximately 2156 mi. 6. (a) 1
65. $V = \frac{r^2\theta h}{2}$, where θ is in radians. (b) 0
66. $V = \frac{1}{2}\theta(r_1^2 - r_2^2)h$, where θ is in radians. (c) undefined
67. $r = \frac{L}{\theta}$. 7. $-\frac{1}{2}$
68. $h = r \cos \frac{\theta}{2}$ 8. $\frac{1}{2}$
69. $d = r \left(1 - \cos \frac{\theta}{2}\right)$ 9. -1
70. $d = \frac{L}{\theta} \left(1 - \cos \frac{\theta}{2}\right)$ 10. -2
71. The area is quadrupled. 11. -2
72. $A = \frac{\pi r^2 \theta}{360}$ 12. $-\sqrt{3}$
13. $-\frac{1}{2}$
14. $-\sqrt{3}$
15. $\frac{\sqrt{2}}{2}$
16. $-\sqrt{2}$
17. $\frac{\sqrt{3}}{2}$
18. $-\frac{1}{2}$
19. $\frac{2\sqrt{3}}{3}$
20. $\frac{2\sqrt{3}}{3}$
21. $-\frac{\sqrt{3}}{3}$
22. $-\frac{\sqrt{2}}{2}$
23. .5736
24. .7314
25. .4068
26. .5397
27. 1.2065
28. .1944

Section 3.3: The Unit Circle and Circular Functions

Connections (page 124)

Answers will vary.

Exercises

1. (a) 1
(b) 0
(c) undefined
2. (a) 0
(b) -1
(c) 0
3. (a) 0
(b) 1
(c) 0
4. (a) 0
(b) -1
(c) 0
5. (a) 0
(b) -1

29. 14.3338
30. 1.0170
31. -1.0460
32. -2.1291
33. -3.8665
34. 1.1848
35. $\cos .8 \approx .7$ 36. $\cos .6 \approx .8$
37. $\sin 2 \approx .9$ 38. $\sin 4 \approx -.75$
39. $\sin 3.8 \approx -.6$ 40. $\cos 3.2 \approx -1.0$
41. $\theta \approx 2.3$ radians or $\theta \approx 4$ radians
42. $\theta \approx 4.4$ radians or $\theta \approx 5.0$ radians
43. $\theta \approx .8$ radians or $\theta \approx 2.4$ radians
44. $\theta \approx 1.3$ radians or $\theta \approx 5.0$ radians
45. negative.
46. negative.
47. negative.
48. positive.
49. positive.
50. negative.
51. $\sin \theta = \frac{\sqrt{2}}{2}$; $\cos \theta = \frac{\sqrt{2}}{2}$;
 $\tan \theta = 1$; $\cot \theta = 1$;
 $\sec \theta = \sqrt{2}$; $\csc \theta = \sqrt{2}$
52. $\sin \theta = \frac{8}{17}$; $\cos \theta = -\frac{15}{17}$; $\tan \theta = -\frac{8}{15}$;
 $\cot \theta = -\frac{15}{8}$; $\sec \theta = -\frac{17}{15}$; $\csc \theta = \frac{17}{8}$
53. $\sin \theta = -\frac{12}{13}$; $\cos \theta = \frac{5}{13}$; $\tan \theta = -\frac{12}{5}$;
 $\cot \theta = -\frac{5}{12}$; $\sec \theta = -\frac{13}{12}$
54. $\sin \theta = y = -\frac{1}{2}$; $\cos \theta = -\frac{\sqrt{3}}{2}$; $\tan \theta = \frac{\sqrt{3}}{3}$;
 $\cot \theta = \sqrt{3}$; $\sec \theta = -\frac{2\sqrt{3}}{3}$; $\csc \theta = -2$
55. .2095
56. .6720
57. 1.4426
58. 1.2799
59. .3887
60. 1.3634
61. $s = \frac{5\pi}{6}$.
62. $s = \frac{2\pi}{3}$.
63. $s = \frac{4\pi}{3}$.
64. $s = \frac{7\pi}{6}$.
65. $s = \frac{7\pi}{4}$.
66. $s = \frac{11\pi}{6}$.
67. $(x, y) = (-.8011, .5985)$
68. $(x, y) = (-.9668, -.2555)$
69. $(x, y) = (.4385, -.8987)$
70. $(x, y) = (-.7259, .6878)$
71. quadrant I.
72. quadrant IV.
73. quadrant II.
74. quadrant III.
75. .9846
76. 2.7824
77. (a) 32.4°
 (b) Answers will vary.
78. 8.6 hr; 15.4 hr
79. (a) 30°
 (b) 60°
 (c) 75°
 (d) 86°
 (e) 86°
 (f) 60°
80. (a) 1°F
 (b) 19°F

- (c) 53°F
- (d) 58°F
- (e) 48°F
- (f) 12°F

Chapter 3: Quiz
(Sections 3.1–3.3)

1. $\frac{5\pi}{4}$
2. $-\frac{11\pi}{6}$
3. 300°
4. -210°
5. 1.5
6. $67,500 \text{ in}^2$
7. $\frac{\sqrt{2}}{2}$
8. $-\frac{1}{2}$
9. 0
10. $\frac{2\pi}{3}$

Section 3.4: Linear and Angular Speed

1. $2\pi \text{ sec}$
2. $2\pi \text{ sec}$
3. (a) $\frac{\pi}{2}$ radians
(b) $10\pi \text{ cm}$
(c) $\frac{5\pi}{3} \text{ cm per sec}$
4. (a) $\frac{2\pi}{5}$ radians
(b) $12\pi \text{ cm}$
(c) $3\pi \text{ cm per sec}$
5. 2π radians
6. $\frac{5\pi}{4}$ radians

7. $\frac{3\pi}{32}$ radian per sec
8. $\frac{\pi}{25}$ radian per sec
9. $\frac{6}{5}$ min
10. 9 min
11. .180311 radian per sec
12. 10.768 radians
13. 8π m per sec
14. $\frac{72\pi}{5}$ cm per sec
15. $\frac{9}{5}$ radians per sec
16. 6 radians per sec
17. 1.83333 radians per sec
18. 9.29755 cm per sec
19. $18\pi \text{ cm}$
20. $\frac{216\pi}{5}$ yd
21. 12 sec
22. 4 sec
23. $\frac{3\pi}{32}$ radian per sec
24. $\frac{\pi}{18}$ radian per sec
25. $\frac{\pi}{6}$ radian per hr
26. 600π radians per min
27. $\frac{\pi}{30}$ radian per min
28. $\frac{\pi}{30}$ radian per sec
29. $\frac{7\pi}{30}$ cm per min
30. $\frac{14\pi}{15}$ mm per sec

31. 168π m per min
32. 1260π cm per min
33. 1500π m per min
34. $112,880\pi$ cm per min
35. 16.6 mph 36.24.62 hr
37. (a) $\frac{2\pi}{365}$ radian
- (b) $\frac{\pi}{4380}$ radian per hr
- (c) about 67,000 mph
38. (a) 2π radians per day; $\frac{\pi}{12}$ radian per hr
- (b) 0
- (c) $12,800\pi$ km per day or about 533π km per hr
- (d) about 28,000 km per day or about 1200 km per hr
39. (a) 3.1 cm per sec
- (b) .24 radian per sec
40. larger pulley:
- $\frac{25\pi}{18}$ radians per sec; smaller pulley:
- $\frac{125\pi}{48}$ radians per sec
41. 3.73 cm 42. about 29 sec
43. 523.6 radians per sec
44. 125 ft per sec
5. $\frac{\pi}{4}$
6. $\frac{2\pi}{3}$
7. $\frac{35\pi}{36}$
8. $\frac{11\pi}{6}$
9. $\frac{40\pi}{9}$
10. $\frac{17\pi}{3}$
11. 225°
12. 162°
13. 480°
14. -216°
15. -110°
16. 756°
17. π in.
18. $\frac{4\pi}{3}$ in.
19. 12π in.
20. 42π in.
21. 35.8 cm
22. 8.77 cm
23. 7.683 cm
24. 2263 in.^2
25. 273 m^2
26. 41 yd
27. 4500 km
28. 12,000 km
29. $\frac{3}{4}$; 1.5 sq units
30. $\frac{1}{2}$ radian; 16 sq units

Chapter 3: Review Exercises

1. the angular measurement of 1 radian is larger than 1° .

2. (a) II.
 (b) III.
 (c) III.
 (d) I.

3. Three of the many possible answers are $1 + 2\pi$, $1 + 4\pi$, and $1 + 6\pi$.

4. $\frac{\pi}{6} + 2n\pi$, where n is any integer

31. (a) $\frac{\pi}{3}$ radians.

(b) 2π in.

32. Answers will vary. Then $2r\left(\frac{\theta}{2}\right)$.

33. $\sqrt{3}$

34. $-\frac{1}{2}$

35. $-\frac{1}{2}$

36. $-\sqrt{3}$

37. 2

38. $\frac{\sqrt{3}}{3}$

39. $\tan 1 > \tan 2$.

40. $\tan 1 > \sin 1$

41. $\sin 2 > \cos 2$.

42. $\cos(\sin 0) > \sin(\cos 0)$.

43. .8660

44. 2.7976

45. .9703

46. -11.4266

47. 1.9513

48. -1.0080

49. .3898

50. 1.3265

51. .5148

52. .9424

53. 1.1054

54. 1.3497

55. $\frac{\pi}{4}$

56. $\frac{2\pi}{3}$

57. $\frac{7\pi}{6}$

58. $\frac{11\pi}{6}$

59. $\frac{15}{32}$ sec

60. 108 radians

61. $\frac{\pi}{20}$ radians per sec

62. $\frac{4\pi}{75}$ radians per sec

63. 285.3 cm

64. 1260π m per sec

65. $\frac{\pi}{36}$ radian per sec.

66. (a) 0; The face of the moon is not visible.

(b) $\frac{1}{2}$; Half the face of the moon is visible.

(c) 1; The face of the moon is completely visible.

(d) $\frac{1}{2}$; Half the face of the moon is visible.

67. (a) $d = hcsc \theta$

(b) $\frac{\pi}{6}$

(c) There is less ultraviolet light when

$\left(\theta = \frac{\pi}{3}\right)$.

68. (a) $y = 2.625 \sin \theta$

(b) $u = 2.625 \cos \theta$

(c) $s = 10.5 \cos \alpha$

(d) $x = 2.625 \cos \theta + 10.5 \cos \alpha$

(e) The maximum velocity of 21.6 mph occurs when $\theta = 4.94$ radians.

Chapter 3 Test

[3.1]

1. $\frac{2\pi}{3}$

2. $-\frac{\pi}{4} - \frac{\pi}{4}$

3. .09
4. 135°
5. -210°
6. 229.18°

[3.2]

7. (a) $\frac{4}{3}$ (b) $15,000 \text{ cm}^2$
8. 2 radians

[3.3]

9. $\frac{\sqrt{2}}{2}$
10. $\frac{\sqrt{3}}{2}$
11. undefined
12. -2
13. 0
14. 0
15. $\sin \frac{7\pi}{6} = -\frac{1}{2}$; $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$;
 $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$
16. sine and cosine: $(-\infty, \infty)$; tangent and secant:
 $\left\{ s \mid s \neq (2n+1)\frac{\pi}{2}, \text{ where } n \text{ is any integer} \right\}$;
cotangent and cosecant:
 $\{s \mid s \neq n\pi, \text{ where } n \text{ is any integer}\}$
17. (a) .9716
(b) $\frac{\pi}{3}$

[3.4]

18. (a) $\frac{2\pi}{3}$ radians
(b) 40π cm
(c) 5π cm per sec
19. approximately 8.278 mi per sec
20. (a) 75 ft
(b) $\frac{\pi}{45}$ radian per sec

Chapter 3: Quantitative Reasoning

1. Triangle RQP is similar to triangle RMO because $m\angle R = m\angle R$ and $m\angle Q = m\angle M$.

2. $r = \frac{c^2}{2b}$

3. $a^2 + b^2 = c^2$; $r = \frac{a^2 + b^2}{2b}$

4. The radius is 5 inches.