

# Chapter 3

## Radian Measure and Circular Functions

### Section 3.1: Radian Measure

- 1.
- 2.
- 3.
4.  $-1$
5.  $-3$
6.  $-2$
7.  $\frac{\pi}{3}$
8.  $\frac{\pi}{6}$
9.  $\frac{\pi}{2}$
10.  $\frac{2\pi}{3}$
11.  $\frac{5\pi}{6}$
12.  $\frac{3\pi}{2}$
13.  $-\frac{5\pi}{3}$
14.  $-\frac{7\pi}{4}$
15.  $\frac{5\pi}{2}$
16.  $\frac{8\pi}{3}$
17.  $10\pi$
18.  $-20\pi$
19. Multiply the degree measure by  $\frac{\pi}{180}$  radian and reduce. Your answer will be in radians. Leave the answer as a multiple of  $\pi$ , unless otherwise directed.
20. Multiply the radian measure by  $\frac{180^\circ}{\pi}$  and reduce. Your answer will be in degrees.
- 21.–24. Answers will vary.
25.  $60^\circ$
26.  $480^\circ$
27.  $315^\circ$
28.  $120^\circ$
29.  $330^\circ$
30.  $675^\circ$
31.  $-30^\circ$
32.  $-288^\circ$
33.  $126^\circ$
34.  $132^\circ$
35.  $-48^\circ$
36.  $-63^\circ$
37.  $153^\circ$
38.  $66^\circ$
39.  $-900^\circ$
40.  $2700^\circ$
41. .68
42. 1.29
43. .742
44. 4.623
45. 2.43
46. 3.05
47. 1.122
48. 1.484
49. .9847
50. 2.140
51. .832391
52. .401675

53.  $114^\circ 35'$

54.  $286^\circ 29'$

55.  $99^\circ 42'$

56.  $175^\circ 20'$

57.  $19^\circ 35'$

58.  $564^\circ 14'$

59.  $-287^\circ 06'$

60.  $-198^\circ 55'$

61. Without the degree symbol on the 30, it is assumed that 30 is measured in radians. Thus, the approximate value of  $\sin 30$  is  $-.98803$ , not  $\frac{1}{2}$ .

62. Answers will vary.

63.  $\frac{\sqrt{3}}{2}$

64.  $\frac{\sqrt{3}}{2}$

65. 1

66.  $\frac{\sqrt{3}}{3}$

67.  $\frac{2\sqrt{3}}{3}$

68.  $\sqrt{2}$

69. 1

70. 1

71.  $-\sqrt{3}$

72.  $-\frac{\sqrt{3}}{3}$

73.  $\frac{1}{2}$

74.  $-\frac{\sqrt{3}}{3}$

75.  $-1$

76.  $-1$

77.  $-\frac{\sqrt{3}}{2}$

78.  $-\frac{\sqrt{3}}{3}$

79.  $\frac{1}{2}$

80.  $\frac{\sqrt{3}}{2}$

81.  $\sqrt{3}$

82.  $-\frac{2\sqrt{3}}{3}$

83. Begin the calculation with the blank next to  $30^\circ$ , and then proceed counterclockwise from there.

$$\frac{\pi}{6}; 45^\circ; \frac{\pi}{3}; 120^\circ; 135^\circ; \frac{5\pi}{6}; \pi; \frac{7\pi}{6}$$

$$\frac{5\pi}{4}; 240^\circ; 300^\circ; \frac{7\pi}{4}; \frac{11\pi}{6}$$

84. (a)  $\frac{\pi}{200}$

(b)  $3.15^\circ; .055$  radian

85. (a)  $4\pi$

(b)  $\frac{2\pi}{3}$

86. (a)  $16\pi$

(b)  $60\pi$

87. (a)  $5\pi$

(b)  $\frac{8\pi}{3}$

88.  $24\pi$

### Section 3.2: Applications of Radian Measure

Answers will vary. The longitude at Greenwich is  $0^\circ$ .

#### Exercises

1.  $2\pi$

2.  $4\pi$

3.  $20\pi$

4. 8

5. 6

6. 8

7. 1  
 8. 1.5  
 9. 2  
 10. Answers will vary.  
 11. 25.8 cm  
 12. 3.08 cm  
 13. 3.61 ft  
 14. 11.9 mi  
 15. 5.05 m.  
 16. 169 cm.  
 17. 55.3 in.  
 18. 71.4 ft  
 19. The length of the arc is doubled.

20.  $s = \frac{\pi r \theta}{180}$

For Exercises 21–26, note that since 6400 has two significant digits and the angles are given to the nearest degree, we can have only two significant digits in the answers.

21. 3500 km  
 22. 1500 km  
 23. 5900 km  
 24. 8800 km  
 25.  $44^\circ$  N  
 26.  $43^\circ$  N  
 27.  $156^\circ$   
 28.  $213^\circ$   
 29.  $38.5^\circ$   
 30.  $82.3^\circ$   
 31. 18.7 cm.  
 32. 29.2 in.  
 33. (a) 11.6 in.  
       (b)  $37^\circ 5'$ .  
 34. 12.7 cm.  
 35. 146 in.  
 36. (a) 39,616 rotations  
       (b) 62.9 mph; Yes  
 37. .20 km  
 38. 850 ft  
 39.  $6\pi$   
 40.  $16\pi$   
 41.  $72\pi$   
 42.  $75\pi$   
 43.  $60^\circ$   
 44.  $240^\circ$   
 45. 1.5  
 46. 1  
 In Exercises 47–54, we will be rounding to the nearest tenth.  
 47. 1116.1 m<sup>2</sup>  
 48. 3744.8 km<sup>2</sup>  
 49. 706.9 ft<sup>2</sup>  
 50. 10,602.9 yd<sup>2</sup>  
 51. 114.0 cm<sup>2</sup>  
 52. 365.3 m<sup>2</sup>  
 53. 1885.0 mi<sup>2</sup>  
 54. 19,085.2 km<sup>2</sup>  
 55. 3.6  
 56. 16 m  
 57. (a)  $13\frac{1}{3}^\circ$ ;  $\frac{2\pi}{27}$   
       (b) 478 ft.  
       (c) 17.7 ft.  
       (d)  $\approx 672$  ft<sup>2</sup>  
 58. 75.4 in.<sup>2</sup>.  
 59. (a) 140 ft  
       (b) 102 ft  
       (c) 622 ft<sup>2</sup>  
 60. 550 m; 1800 m  
 61. 1900 yd<sup>2</sup>  
 62. 1.15 mi

63. radius: 3950 mi.; circumference: 24,800 mi. (c) 0
64. approximately 2156 mi. 6. (a) 1
65.  $V = \frac{r^2\theta h}{2}$ , where  $\theta$  is in radians. (b) 0
66.  $V = \frac{1}{2}\theta(r_1^2 - r_2^2)h$ , where  $\theta$  is in radians. (c) undefined
67.  $r = \frac{L}{\theta}$ . 7.  $-\frac{1}{2}$
68.  $h = r \cos \frac{\theta}{2}$  8.  $\frac{1}{2}$
69.  $d = r \left(1 - \cos \frac{\theta}{2}\right)$  9. -1
70.  $d = \frac{L}{\theta} \left(1 - \cos \frac{\theta}{2}\right)$  10. -2
71. The area is quadrupled. 11. -2
72.  $A = \frac{\pi r^2 \theta}{360}$  12.  $-\sqrt{3}$
13.  $-\frac{1}{2}$
14.  $-\sqrt{3}$
15.  $\frac{\sqrt{2}}{2}$
16.  $-\sqrt{2}$
17.  $\frac{\sqrt{3}}{2}$
18.  $-\frac{1}{2}$
19.  $\frac{2\sqrt{3}}{3}$
20.  $\frac{2\sqrt{3}}{3}$
21.  $-\frac{\sqrt{3}}{3}$
22.  $-\frac{\sqrt{2}}{2}$
23. .5736
24. .7314
25. .4068
26. .5397
27. 1.2065
28. .1944

### Section 3.3: The Unit Circle and Circular Functions

#### Connections (page 124)

Answers will vary.

#### Exercises

1. (a) 1  
(b) 0  
(c) undefined
2. (a) 0  
(b) -1  
(c) 0
3. (a) 0  
(b) 1  
(c) 0
4. (a) 0  
(b) -1  
(c) 0
5. (a) 0  
(b) -1

29. 14.3338
30. 1.0170
31. -1.0460
32. -2.1291
33. -3.8665
34. 1.1848
35.  $\cos .8 \approx .7$       36.  $\cos .6 \approx .8$
37.  $\sin 2 \approx .9$       38.  $\sin 4 \approx -.75$
39.  $\sin 3.8 \approx -.6$       40.  $\cos 3.2 \approx -1.0$
41.  $\theta \approx 2.3$  radians or  $\theta \approx 4$  radians
42.  $\theta \approx 4.4$  radians or  $\theta \approx 5.0$  radians
43.  $\theta \approx .8$  radians or  $\theta \approx 2.4$  radians
44.  $\theta \approx 1.3$  radians or  $\theta \approx 5.0$  radians
45. negative.
46. negative.
47. negative.
48. positive.
49. positive.
50. negative.
51.  $\sin \theta = \frac{\sqrt{2}}{2}$ ;  $\cos \theta = \frac{\sqrt{2}}{2}$ ;  
 $\tan \theta = 1$ ;  $\cot \theta = 1$ ;  
 $\sec \theta = \sqrt{2}$ ;  $\csc \theta = \sqrt{2}$
52.  $\sin \theta = \frac{8}{17}$ ;  $\cos \theta = -\frac{15}{17}$ ;  $\tan \theta = -\frac{8}{15}$ ;  
 $\cot \theta = -\frac{15}{8}$ ;  $\sec \theta = -\frac{17}{15}$ ;  $\csc \theta = \frac{17}{8}$
53.  $\sin \theta = -\frac{12}{13}$ ;  $\cos \theta = \frac{5}{13}$ ;  $\tan \theta = -\frac{12}{5}$ ;  
 $\cot \theta = -\frac{5}{12}$ ;  $\sec \theta = -\frac{13}{12}$
54.  $\sin \theta = y = -\frac{1}{2}$ ;  $\cos \theta = -\frac{\sqrt{3}}{2}$ ;  $\tan \theta = \frac{\sqrt{3}}{3}$ ;  
 $\cot \theta = \sqrt{3}$ ;  $\sec \theta = -\frac{2\sqrt{3}}{3}$ ;  $\csc \theta = -2$
55. .2095
56. .6720
57. 1.4426
58. 1.2799
59. .3887
60. 1.3634
61.  $s = \frac{5\pi}{6}$ .
62.  $s = \frac{2\pi}{3}$ .
63.  $s = \frac{4\pi}{3}$ .
64.  $s = \frac{7\pi}{6}$ .
65.  $s = \frac{7\pi}{4}$ .
66.  $s = \frac{11\pi}{6}$ .
67.  $(x, y) = (-.8011, .5985)$
68.  $(x, y) = (-.9668, -.2555)$
69.  $(x, y) = (.4385, -.8987)$
70.  $(x, y) = (-.7259, .6878)$
71. quadrant I.
72. quadrant IV.
73. quadrant II.
74. quadrant III.
75. .9846
76. 2.7824
77. (a)  $32.4^\circ$   
 (b) Answers will vary.
78. 8.6 hr; 15.4 hr
79. (a)  $30^\circ$   
 (b)  $60^\circ$   
 (c)  $75^\circ$   
 (d)  $86^\circ$   
 (e)  $86^\circ$   
 (f)  $60^\circ$
80. (a)  $1^\circ\text{F}$   
 (b)  $19^\circ\text{F}$

- (c)  $53^\circ\text{F}$
- (d)  $58^\circ\text{F}$
- (e)  $48^\circ\text{F}$
- (f)  $12^\circ\text{F}$

**Chapter 3: Quiz**  
(Sections 3.1–3.3)

1.  $\frac{5\pi}{4}$
2.  $-\frac{11\pi}{6}$
3.  $300^\circ$
4.  $-210^\circ$
5. 1.5
6.  $67,500 \text{ in}^2$
7.  $\frac{\sqrt{2}}{2}$
8.  $-\frac{1}{2}$
9. 0
10.  $\frac{2\pi}{3}$
7.  $\frac{3\pi}{32}$  radian per sec
8.  $\frac{\pi}{25}$  radian per sec
9.  $\frac{6}{5}$  min
10. 9 min
11. .180311 radian per sec
12. 10.768 radians
13.  $8\pi$  m per sec
14.  $\frac{72\pi}{5}$  cm per sec
15.  $\frac{9}{5}$  radians per sec
16. 6 radians per sec
17. 1.83333 radians per sec
18. 9.29755 cm per sec
19.  $18\pi$  cm
20.  $\frac{216\pi}{5}$  yd
21. 12 sec
22. 4 sec

**Section 3.4: Linear and Angular Speed**

1.  $2\pi$  sec
2.  $2\pi$  sec
3. (a)  $\frac{\pi}{2}$  radians
- (b)  $10\pi$  cm
- (c)  $\frac{5\pi}{3}$  cm per sec
4. (a)  $\frac{2\pi}{5}$  radians
- (b)  $12\pi$  cm
- (c)  $3\pi$  cm per sec
5.  $2\pi$  radians
6.  $\frac{5\pi}{4}$  radians
23.  $\frac{3\pi}{32}$  radian per sec
24.  $\frac{\pi}{18}$  radian per sec
25.  $\frac{\pi}{6}$  radian per hr
26.  $600\pi$  radians per min
27.  $\frac{\pi}{30}$  radian per min
28.  $\frac{\pi}{30}$  radian per sec
29.  $\frac{7\pi}{30}$  cm per min
30.  $\frac{14\pi}{15}$  mm per sec

31.  $168\pi$  m per min  
 32.  $1260\pi$  cm per min  
 33.  $1500\pi$  m per min  
 34.  $112,880\pi$  cm per min  
 35. 16.6 mph 36.24.62 hr  
 37. (a)  $\frac{2\pi}{365}$  radian  
 (b)  $\frac{\pi}{4380}$  radian per hr  
 (c) about 67,000 mph  
 38. (a)  $2\pi$  radians per day;  $\frac{\pi}{12}$  radian per hr  
 (b) 0  
 (c)  $12,800\pi$  km per day or about  $533\pi$  km per hr  
 (d) about 28,000 km per day or about 1200 km per hr  
 39. (a) 3.1 cm per sec  
 (b) .24 radian per sec  
 40. larger pulley:  
 $\frac{25\pi}{18}$  radians per sec; smaller pulley:  
 $\frac{125\pi}{48}$  radians per sec  
 41. 3.73 cm 42. about 29 sec  
 43. 523.6 radians per sec  
 44. 125 ft per sec
- Chapter 3: Review Exercises**
1. the angular measurement of 1 radian is larger than  $1^\circ$ .  
 2. (a) II.  
 (b) III.  
 (c) III.  
 (d) I.  
 3. Three of the many possible answers are  $1 + 2\pi$ ,  $1 + 4\pi$ , and  $1 + 6\pi$ .  
 4.  $\frac{\pi}{6} + 2n\pi$ , where  $n$  is any integer
5.  $\frac{\pi}{4}$   
 6.  $\frac{2\pi}{3}$   
 7.  $\frac{35\pi}{36}$   
 8.  $\frac{11\pi}{6}$   
 9.  $\frac{40\pi}{9}$   
 10.  $\frac{17\pi}{3}$   
 11.  $225^\circ$   
 12.  $162^\circ$   
 13.  $480^\circ$   
 14.  $-216^\circ$   
 15.  $-110^\circ$   
 16.  $756^\circ$   
 17.  $\pi$  in.  
 18.  $\frac{4\pi}{3}$  in.  
 19.  $12\pi$  in.  
 20.  $42\pi$  in.  
 21. 35.8 cm  
 22. 8.77 cm  
 23. 7.683 cm  
 24.  $2263 \text{ in.}^2$   
 25.  $273 \text{ m}^2$   
 26. 41 yd  
 27. 4500 km  
 28. 12,000 km  
 29.  $\frac{3}{4}$ ; 1.5 sq units  
 30.  $\frac{1}{2}$  radian; 16 sq units

31. (a)  $\frac{\pi}{3}$  radians.

(b)  $2\pi$  in.

32. Answers will vary. Then  $2r\left(\frac{\theta}{2}\right)$ .

33.  $\sqrt{3}$

34.  $-\frac{1}{2}$

35.  $-\frac{1}{2}$

36.  $-\sqrt{3}$

37. 2

38.  $\frac{\sqrt{3}}{3}$

39.  $\tan 1 > \tan 2$ .

40.  $\tan 1 > \sin 1$

41.  $\sin 2 > \cos 2$ .

42.  $\cos(\sin 0) > \sin(\cos 0)$ .

43. .8660

44. 2.7976

45. .9703

46. -11.4266

47. 1.9513

48. -1.0080

49. .3898

50. 1.3265

51. .5148

52. .9424

53. 1.1054

54. 1.3497

55.  $\frac{\pi}{4}$

56.  $\frac{2\pi}{3}$

57.  $\frac{7\pi}{6}$

58.  $\frac{11\pi}{6}$

59.  $\frac{15}{32}$  sec

60. 108 radians

61.  $\frac{\pi}{20}$  radians per sec

62.  $\frac{4\pi}{75}$  radians per sec

63. 285.3 cm

64.  $1260\pi$  m per sec

65.  $\frac{\pi}{36}$  radian per sec.

66. (a) 0; The face of the moon is not visible.

(b)  $\frac{1}{2}$ ; Half the face of the moon is visible.

(c) 1; The face of the moon is completely visible.

(d)  $\frac{1}{2}$ ; Half the face of the moon is visible.

67. (a)  $d = hcsc \theta$

(b)  $\frac{\pi}{6}$

(c) There is less ultraviolet light when

$\left(\theta = \frac{\pi}{3}\right)$ .

68. (a)  $y = 2.625 \sin \theta$

(b)  $u = 2.625 \cos \theta$

(c)  $s = 10.5 \cos \alpha$

(d)  $x = 2.625 \cos \theta + 10.5 \cos \alpha$

(e) The maximum velocity of 21.6 mph occurs when  $\theta = 4.94$  radians.

**Chapter 3 Test**

[3.1]

1.  $\frac{2\pi}{3}$

2.  $-\frac{\pi}{4} - \frac{\pi}{4}$

3. .09
4.  $135^\circ$
5.  $-210^\circ$
6.  $229.18^\circ$

[3.2]

7. (a)  $\frac{4}{3}$  (b)  $15,000 \text{ cm}^2$
8. 2 radians

[3.3]

9.  $\frac{\sqrt{2}}{2}$
10.  $\frac{\sqrt{3}}{2}$
11. undefined
12. -2
13. 0
14. 0
15.  $\sin \frac{7\pi}{6} = -\frac{1}{2}$ ;  $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ ;  
 $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$
16. sine and cosine:  $(-\infty, \infty)$ ; tangent and secant:  
 $\left\{ s \mid s \neq (2n+1)\frac{\pi}{2}, \text{ where } n \text{ is any integer} \right\}$ ;  
cotangent and cosecant:  
 $\{s \mid s \neq n\pi, \text{ where } n \text{ is any integer}\}$
17. (a) .9716  
(b)  $\frac{\pi}{3}$

[3.4]

18. (a)  $\frac{2\pi}{3}$  radians  
(b)  $40\pi$  cm  
(c)  $5\pi$  cm per sec
19. approximately 8.278 mi per sec
20. (a) 75 ft  
(b)  $\frac{\pi}{45}$  radian per sec

### Chapter 3: Quantitative Reasoning

1. Triangle RQP is similar to triangle RMO because  $m\angle R = m\angle R$  and  $m\angle Q = m\angle M$ .

2.  $r = \frac{c^2}{2b}$

3.  $a^2 + b^2 = c^2$ ;  $r = \frac{a^2 + b^2}{2b}$

4. The radius is 5 inches.