

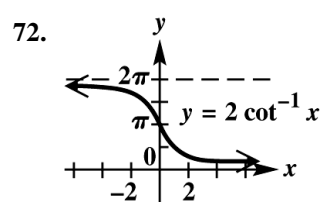
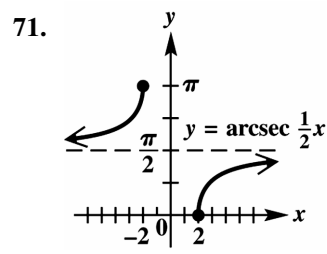
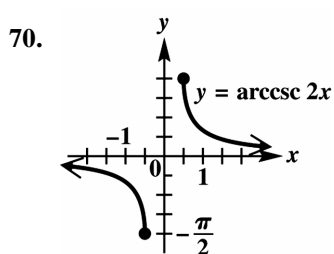
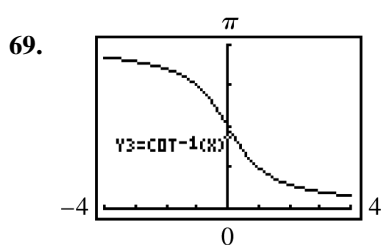
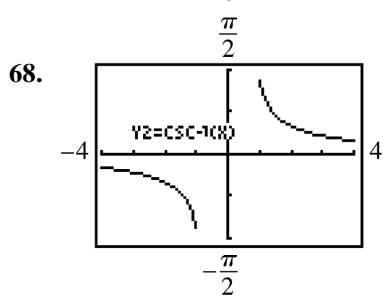
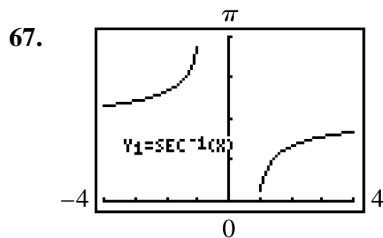
# Chapter 6

## Inverse Circular Functions and Trigonometric Equations

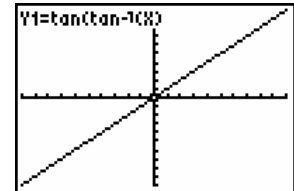
### Section 6.1: Inverse Circular Functions

- one-to-one.
- range
- domain
- $\left(1, \frac{\pi}{4}\right)$ ;  $y = \tan^{-1} x$ , or  $y = \arctan x$
- $\pi$ .
- Sketch the reflection of the graph of  $f$  across the line  $y = x$ .
- (a)  $[-1, 1]$   
(b)  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$   
(c) increasing  
(d)  $-2$  is not in the domain.
- (a)  $[-1, 1]$   
(b)  $[0, \pi]$   
(c) decreasing  
(d)  $-\frac{4\pi}{3}$  is not in the range.
- (a)  $(-\infty, \infty)$   
(b)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
(c) increasing  
(d) no
- (a)  $(-\infty, -1] \cup [1, \infty); \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$   
(b)  $(-\infty, -1] \cup [1, \infty); \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$   
(c)  $(-\infty, \infty); (0, \pi)$
- $\cos^{-1} \frac{1}{a}$
- Find  $\tan^{-1} \frac{1}{a} + \pi$  or  $\tan^{-1} \frac{1}{a} + 180^\circ$ .
- $y = 0$ .
- $\pi$
- $-\frac{\pi}{2}$
- $0$
- $\frac{\pi}{2}$
- $\frac{\pi}{4}$
- $\frac{5\pi}{6}$
- $\frac{3\pi}{4}$
- $-\frac{\pi}{6}$
- $\frac{\pi}{6}$
- $0$
- $-45^\circ$
- $-60^\circ$
- $120^\circ$
- $120^\circ$
- $60^\circ$
- $\sin^{-1} 2$  does not exist.
- $\cos^{-1}(-2)$  does not exist.
- $-7.6713835^\circ$
- $97.671207^\circ$
- $113.500970^\circ$
- $51.1691219^\circ$
- $30.987961^\circ$
- $y = \frac{\pi}{4}$ .
- $-\frac{\pi}{4}$
- $\frac{\pi}{3}$
- $-\frac{\pi}{3}$
- $-\frac{\pi}{4}$
- $\frac{2\pi}{3}$
- $-\frac{\pi}{4}$
- $\frac{2\pi}{3}$
- $-\frac{\pi}{4}$
- $\frac{3\pi}{4}$
- $\frac{5\pi}{6}$
- $\frac{\pi}{4}$
- No
- $120^\circ$
- $-45^\circ$
- $-30^\circ$
- $-90^\circ$
- $60^\circ$
- $60^\circ$

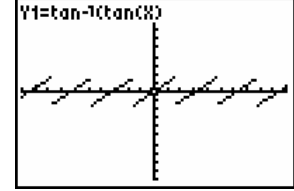
- 52. 29.506181°
- 53. 121.267893°
- 54. 160.172137°
- 55. -82.678329°
- 56. 101.267354°
- 57. .83798122
- 58. .96012698
- 59. 2.3154725
- 60. 2.4605221
- 61. 1.1900238
- 62. 1.1082303
- 63. 1.9033723
- 64. 1.3607651
- 65. 3.1144804
- 66. .78357295



- 73. 1.003 is not in the domain of  $y = \sin^{-1} x$ . (Alternatively, you could state that 1.003 is not in the range of  $y = \sin x$ .)
- 74. In each case the result is  $x$ . The graph is a straight line bisecting quadrants I and III (i.e., the line  $y = x$ ).
- 75. It is the graph of  $y = x$ .



- 76. It does not agree because the range of the inverse tangent function is  $(-\frac{\pi}{2}, \frac{\pi}{2})$ , not  $(-\infty, \infty)$ , as was the case in Exercise 74.



- 77.  $\frac{\sqrt{7}}{3}$
- 78.  $\frac{\sqrt{15}}{4}$
- 79.  $\frac{\sqrt{5}}{5}$
- 80.  $\frac{5\sqrt{6}}{12}$

81.  $\frac{120}{169}$

82.  $\frac{7}{8}$

83.  $-\frac{7}{25}$

84.  $-\frac{\sqrt{15}}{7}$

85.  $\sin \theta = \frac{2\sqrt{6}}{5}$ . Then,  $\frac{4\sqrt{6}}{25}$

86.  $-\frac{3}{5}$

87. 2

88.  $\sqrt{2}$

89.  $\frac{63}{65}$

90.  $-\frac{16}{65}$

91.  $\frac{\sqrt{10} - 3\sqrt{30}}{20}$

92.  $\frac{48 + 25\sqrt{3}}{39}$

93. .894427191

94. .9682458366

95. .1234399811

96. .716386406

97.  $\sqrt{1-u^2}$

98.  $\frac{\sqrt{1-u^2}}{u}$

99.  $\sqrt{1-u^2}$

100.  $\frac{\sqrt{1-u^2}}{u}$

101.  $\frac{4\sqrt{u^2-4}}{u^2}$

102.  $\frac{u^2-9}{u^2+9}$

103.  $\frac{u\sqrt{2}}{2}$

104.  $\frac{\sqrt{u^2+5}}{u}$

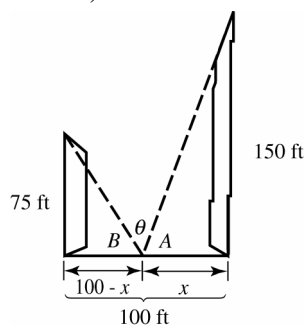
105.  $\frac{2\sqrt{4-u^2}}{4-u^2}$

106.  $\frac{3\sqrt{9-u^2}}{9-u^2}$

107. (a)  $45^\circ$

(b)  $\theta = 45^\circ$ .

108. Let  $A$  be the angle to the “right” of  $\theta$  and let  $B$  be the angle to the “left” of  $\theta$ . Then  $A + \theta + B = \pi$  (since the angles sum to  $\pi$  radians) and  $\theta = \pi - A - B$ .



$$\tan A = \frac{150}{x}, \text{ so } A = \arctan\left(\frac{150}{x}\right) \text{ and}$$

$$\tan B = \frac{75}{100-x}, \text{ so } B = \arctan\left(\frac{75}{100-x}\right).$$

$$\text{As a result, } \theta = \pi - B - A \Rightarrow$$

$$\theta = \pi - \arctan\left(\frac{75}{100-x}\right) - \arctan\left(\frac{150}{x}\right).$$

109. (a)  $18^\circ$

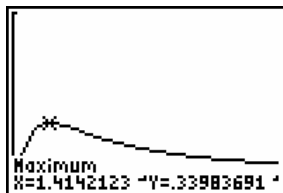
(b)  $18^\circ$

(c)  $15^\circ$

$$(d) \tan(\theta + \alpha) = \frac{1+1}{x} = \frac{2}{x} \text{ and } \tan \alpha = \frac{1}{x}$$

$$\begin{aligned} \tan(\theta + \alpha) &= \frac{\tan \theta + \tan \alpha}{1 - \tan \theta \tan \alpha} \\ \frac{2}{x} &= \frac{\tan \theta + \frac{1}{x}}{1 - \tan \theta \left(\frac{1}{x}\right)} \\ \frac{2}{x} &= \frac{x \tan \theta + 1}{x - \tan \theta} \\ 2(x - \tan \theta) &= x(x \tan \theta + 1) \\ 2x - 2 \tan \theta &= x^2 \tan \theta + x \\ 2x - x &= x^2 \tan \theta + 2 \tan \theta \\ x &= \tan \theta (x^2 + 2) \\ \tan \theta &= \frac{x}{x^2 + 2} \\ \theta &= \tan^{-1} \left( \frac{x}{x^2 + 2} \right) \end{aligned}$$

- (e) 1.4142151 m. (Note: Due to the computational routine, there may be a discrepancy in the last few decimal places.)



(f)  $\sqrt{2}$

110. 44.7%.

## Section 6.2: Trigonometric Equations I

- Solve the linear equation for  $\cot x$ .
- Solve the linear equation for  $\sin x$ .
- Solve the quadratic equation for  $\sec x$  by factoring.
- Solve the quadratic equation for  $\cos x$  by the factoring.
- Solve the quadratic equation for  $\sin x$  using the quadratic formula.
- Solve the quadratic for  $\tan x$  using the quadratic formula.
- Use the identity to rewrite as an equation with one trigonometric function.

8. Use an identity to rewrite as an equation with one trigonometric function.

9.  $-30^\circ$  is not in the interval  $[0^\circ, 360^\circ)$ .

10. To show that  $\left\{0, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$  is not the correct solution set to the equation  $\sin x = 1 - \cos x$ , show that at least one element of the set is not a solution.

Check  $x = 0$ .

$$\sin x = 1 - \cos x$$

$$\sin 0 = 1 - \cos 0 \quad ?$$

$$0 = 1 - 1 \quad ?$$

$$0 = 0 \quad \text{True}$$

$x = 0$  is a solution.

Check  $x = \frac{\pi}{2}$ .

$$\sin x = 1 - \cos x$$

$$\sin \frac{\pi}{2} = 1 - \cos \frac{\pi}{2} \quad ?$$

$$1 = 1 - 0 \quad ?$$

$$1 = 1 \quad \text{True}$$

$x = \frac{\pi}{2}$  is a solution.

Check  $x = \frac{3\pi}{2}$ .

$$\sin x = 1 - \cos x$$

$$\sin \frac{3\pi}{2} = 1 - \cos \frac{3\pi}{2} \quad ?$$

$$-1 = 1 - 0 \quad ?$$

$$-1 = 1 \quad \text{False}$$

$x = \frac{3\pi}{2}$  is not a solution.

In general, when you square both sides of an equation or raise both sides of an equation to an even power, you must check all solutions in order to eliminate any extraneous solutions.

11.  $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

12.  $\left\{\frac{\pi}{2}\right\}$

13.  $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

14.  $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

15.  $\emptyset$

16.  $\emptyset$

17.  $\left\{\frac{\pi}{4}, \frac{2\pi}{3}, \frac{5\pi}{4}, \frac{5\pi}{3}\right\}$

18.  $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

19.  $\{\pi\}$

20.  $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

21.  $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

22.  $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

23.  $\{30^\circ, 210^\circ, 240^\circ, 300^\circ\}$

24.  $\{0^\circ, 45^\circ, 225^\circ\}$

25.  $\{90^\circ, 210^\circ, 330^\circ\}$

26.  $\{60^\circ, 135^\circ, 240^\circ, 315^\circ\}$

27.  $\{45^\circ, 135^\circ, 225^\circ, 315^\circ\}$

28.  $\{0^\circ, 180^\circ\}$

29.  $\{45^\circ, 225^\circ\}$

30.  $\{90^\circ, 270^\circ\}$

31.  $\{0^\circ, 30^\circ, 150^\circ, 180^\circ\}$

32.  $\{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$

33.  $\{0^\circ, 45^\circ, 135^\circ, 180^\circ, 225^\circ, 315^\circ\}$

34.  $\{45^\circ, 135^\circ, 225^\circ, 315^\circ\}$

35.  $\{53.6^\circ, 126.4^\circ, 187.9^\circ, 352.1^\circ\}$

36.  $\{78.0^\circ, 282.0^\circ\}$

37.  $\{106.3^\circ, 149.6^\circ, 286.3^\circ, 329.6^\circ\}$

38.  $\{38.4^\circ, 104.8^\circ, 218.4^\circ, 284.8^\circ\}$

39.  $\emptyset$

40.  $\{68.5^\circ, 291.5^\circ\}$

41.  $\{57.7^\circ, 159.2^\circ\}$

42.  $\{114.3^\circ, 335.7^\circ\}$

In Exercises 43–46, if you are using a calculator, make sure it is in radian mode.

43.  $\{.8751 + 2n\pi, 2.2665 + 2n\pi, 3.5908 + 2n\pi,$   
and  $5.8340 + 2n\pi$ , where  $n$  is any integer}.

44.  $\left\{\frac{\pi}{3} + 2n\pi, \pi + 2n\pi,$   
and  $\frac{5\pi}{3} + 2n\pi$ , where  $n$  is any integer }.

45.  $\left\{\frac{\pi}{3} + n\pi$  and  $\frac{2\pi}{3} + n\pi$ , where  $n$  is any integer }.

46.  $\left\{\frac{2\pi}{3} + 2n\pi$  and  $\frac{4\pi}{3} + 2n\pi$ , where  $n$  is any integer }.

47.  $\{33.6^\circ + 360^\circ n$  and  $326.4^\circ + 360^\circ n$ , where  $n$  is any integer }.

48.  $\{90^\circ + 360^\circ n, 221.8^\circ + 360^\circ n,$   
and  $318.2^\circ + 360^\circ n$ , where  $n$  is any integer }.

49.  $\{45^\circ + n \cdot 180^\circ$  and  $108.4^\circ + n \cdot 180^\circ$ , where  $n$  is any integer }.

50.  $\{71.6^\circ + n \cdot 180^\circ$  and  $135^\circ + n \cdot 180^\circ$ , where  $n$  is any integer }.

51.  $\{.6806, 1.4159\}$

52.  $\{0, .3760\}$

53. (a) .00164 and .00355.

(b) [.00164, .00355].

(c) outward

54.  $14^\circ$

55. (a)  $\frac{1}{4}$  sec

(b)  $\frac{1}{6}$  sec

(c) .21 sec

56. (a) 2 sec

(b)  $3\frac{1}{3}$  sec

57. (a) One such value is  $\frac{\pi}{3}$ .

(b) One such value is  $\frac{\pi}{4}$ .

58. In the second line of the “solution”, both sides of the equation were divided by  $\sin x$ . Instead of dividing by  $\sin x$ , one should have factored  $\sin x$  from  $\sin^2 x - \sin x$ . In the process of dividing both sides by  $\sin x$ , the solutions of  $x = 0$  and  $x = \pi$  were eliminated.

**Section 6.3: Trigonometric Equations II**

1.  $\left\{ \frac{\pi}{3}, \pi, \frac{4\pi}{3} \right\}$

2.  $\left\{ \frac{\pi}{8}, \frac{5\pi}{6}, \frac{5\pi}{4} \right\}$

3.  $\{60^\circ, 210^\circ, 240^\circ, 310^\circ\}$ .

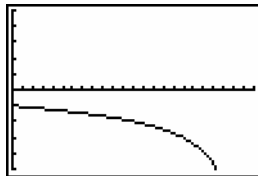
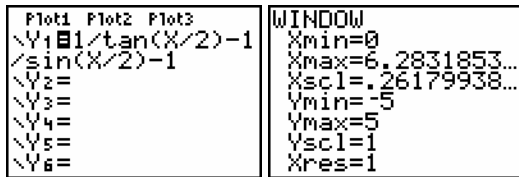
4.  $\{135^\circ, 180^\circ, 225^\circ, 270^\circ\}$ .

5.  $\frac{\tan 2\theta}{2} \neq \tan \theta$  for all values of  $\theta$ .

6. If  $\cot \frac{x}{2} - \csc \frac{x}{2} - 1 = 0$  has no solutions in the interval  $[0, 2\pi)$ , then the graph of

$y = \cot \frac{x}{2} - \csc \frac{x}{2} - 1$  will have no  $x$ -intercepts

in this same interval.



7.  $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

8.  $\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

9.  $\left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

10.  $\left\{ 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

11.  $\left\{ \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18} \right\}$

12.  $\left\{ \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18} \right\}$

13.  $\left\{ \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8} \right\}$

14.  $\left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$

15.  $\left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$

16.  $\left\{ 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \right\}$

17.  $\left\{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$

18.  $\left\{ 0, \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$

19.  $\emptyset$

20.  $\emptyset$

21.  $\left\{ \frac{\pi}{2} \right\}$

22.  $\{0\}$

23.  $\left\{ \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$

24.  $\left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$

25.  $\{15^\circ, 45^\circ, 135^\circ, 165^\circ, 255^\circ, 285^\circ\}$

26.  $\{75^\circ, 105^\circ, 255^\circ, 285^\circ\}$

27.  $\{0^\circ\}$

28.  $\{180^\circ\}$

29.  $\{120^\circ, 240^\circ\}$

30.  $\{300^\circ\}$

31.  $\{30^\circ, 150^\circ, 270^\circ\}$

32.  $\{60^\circ, 90^\circ, 270^\circ, 300^\circ\}$
33.  $\{0^\circ + 360^\circ n, 30^\circ + 360^\circ n, 150^\circ + 360^\circ n, 180^\circ + 360^\circ n,$   
where  $n$  is any integer} or  
 $\{180^\circ n, 30^\circ + 360^\circ n, 150^\circ + 360^\circ n,$   
where  $n$  is any integer}
34.  $\{45^\circ + 360^\circ n, 90^\circ + 360^\circ n, 225^\circ + 360^\circ n, 270^\circ + 360^\circ n,$  where  $n$   
is any integer} or  $\{45^\circ + 180^\circ n, 90^\circ + 180^\circ n,$   
where  $n$  is any integer}

35.  $\{60^\circ + 360^\circ n, 300^\circ + 360^\circ n,$   
where  $n$  is any integer}

36.  $\{70.5^\circ + 360^\circ n, 289.5^\circ + 360^\circ n,$   
where  $n$  is any integer}

37.  $\{11.8^\circ + 180^\circ n, 78.2^\circ + 180^\circ n,$   
where  $n$  is any integer}

38.  $\{22.5^\circ + 180^\circ n, 112.5^\circ + 180^\circ n,$   
where  $n$  is any integer}

39.  $\{30^\circ + 180^\circ n, 90^\circ + 180^\circ n, 150^\circ + 180^\circ n,$   
where  $n$  is any integer}

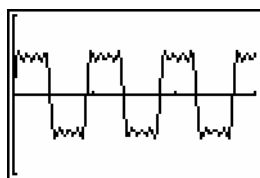
40.  $\{180^\circ n, 60^\circ + 360^\circ n, 300^\circ + 360^\circ n,$   
where  $n$  is any integer}

41.  $\{1.2802\}$

42.  $\{.6919, 2.0820\}$

43. (a) For  $x = t$   

$$P(t) = .003 \sin 220\pi t + \frac{.003}{3} \sin 660\pi t + \frac{.003}{5} \sin 1100\pi t + \frac{.003}{7} \sin 1540\pi t$$



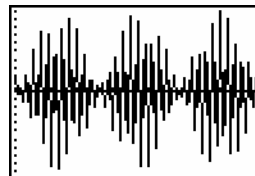
(b) The graph is periodic, and the wave has “jagged square” tops and bottoms.

(c) This occurs for the time intervals  $(.0045, .0091), (.0136, .0182), (.0227, .0273).$

44. (a) 3 beats per sec

For  $x = t$   

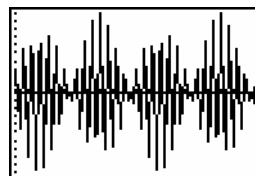
$$P(t) = .005 \sin 440\pi t + .005 \sin 446\pi t$$



(b) 4 beats per sec

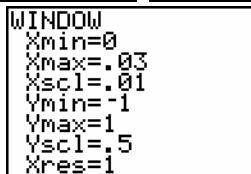
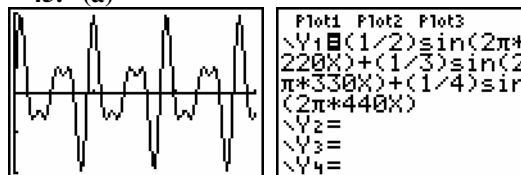
For  $x = t$   

$$P(t) = .005 \sin 440\pi t + .005 \sin 432\pi t$$



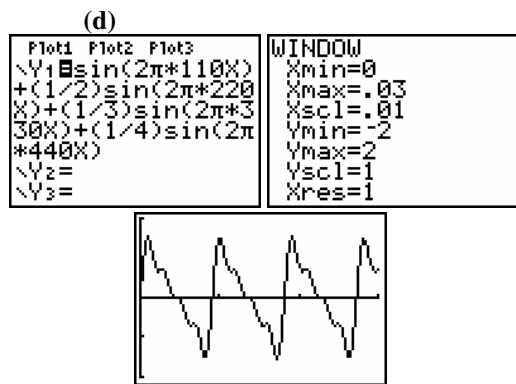
(c) The number of beats is equal to the absolute value of the difference in the frequencies of the two tones.

45. (a)



(b) .0007586, .009849, .01894, .02803

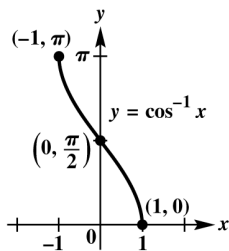
(c) 110 Hz



46. (a)  $x = 91.25$  means about 91.3 days after March 21, on June 20.  
 (b)  $x = 273.75$  means about 273.8 days after March 21, on December 19.  
 (c)  $x = 318.8$  means about 318.8 days after March 21, on February 2.
47. .001sec  
 48. .0007 sec  
 49. .004sec  
 50. .0014 sec

### Chapter 6 Quiz (Sections 6.1–6.3)

1.  $[-1, 1]; [0, \pi]$



2. (a)  $-\frac{\pi}{4}$   
 (b)  $\frac{\pi}{3}$   
 (c)  $\frac{5\pi}{6}$
3. (a)  $22.568922^\circ$  (b)  $137.431085^\circ$
4. (a)  $\frac{5\sqrt{41}}{41}$  (b)  $\frac{\sqrt{3}}{2}$
5.  $\{60^\circ, 120^\circ\}$

6.  $\{60^\circ, 180^\circ, 300^\circ\}$   
 7.  $\{.6089, 1.3424, 3.7505, 4.4840\}$   
 8.  $\left\{\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}\right\}$   
 9.  $\left\{\frac{5\pi}{3} + 4n\pi, \frac{7\pi}{3} + 4n\pi\right\}$   
 10. (a) 0sec  
 (b) .20sec

### Section 6.4: Equations Involving Inverse Trigonometric Functions

1. C  
 2. A  
 3. C  
 4. C  
 5.  $x = \arccos \frac{y}{5}$   
 6.  $x = \arcsin 4y$   
 7.  $x = \frac{1}{3} \operatorname{arccot} 2y$   
 8.  $x = \operatorname{arcsec} 12y$   
 9.  $x = \frac{1}{2} \arctan \frac{y}{3}$   
 10.  $x = 2 \arcsin \frac{y}{3}$   
 11.  $x = 4 \arccos \frac{y}{6}$   
 12.  $x = 3 \arcsin(-y)$   
 13.  $x = \frac{1}{5} \arccos\left(-\frac{y}{2}\right)$   
 14.  $x = \frac{1}{5} \operatorname{arccot} \frac{y}{3}$   
 15.  $x = -3 + \arccos y$   
 16.  $x = \frac{1}{2}(1 + \arctan y)$   
 17.  $x = \arcsin(y + 2)$

18.  $x = \operatorname{arccot}(y-1)$

19.  $x = \arcsin\left(\frac{y+4}{2}\right)$

20.  $x = \arccos\left(\frac{y-4}{3}\right)$

21.  $x = \frac{1}{2} \sec^{-1}\left(\frac{y-\sqrt{2}}{3}\right)$

22.  $x = 2 \csc^{-1}\left(\frac{y+\sqrt{3}}{2}\right)$

23. First,  $\sin x - 2 \neq \sin(x-2)$ . If you think of the graph of  $y = \sin x - 2$ , this represents the graph of  $f(x) = \sin x$ , shifted 2 units down. If you think of the graph of  $y = \sin(x-2)$ , this represents the graph of  $f(x) = \sin x$ , shifted 2 units right.

24.  $\cos^{-1} 2$  doesn't exist since there is no value  $x$  such that  $\cos x = 2$ .

25.  $\left\{-\frac{\sqrt{2}}{2}\right\}$

26.  $\left\{-\frac{\sqrt{3}}{2}\right\}$

27.  $\{-2\sqrt{2}\}$

28.  $\emptyset$

29.  $\{\pi - 3\}$

30.  $\left\{\frac{3\sqrt{3} + 2\pi}{6}\right\}$

31.  $\left\{\frac{3}{5}\right\}$

32.  $\left\{\frac{12}{5}\right\}$

33.  $\left\{\frac{4}{5}\right\}$

34.  $\left\{\frac{3}{4}\right\}$

35.  $\{0\}$

36.  $\left\{\frac{\sqrt{3}}{2}\right\}$

37.  $\left\{\frac{1}{2}\right\}$

38.  $\emptyset$

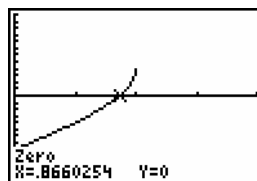
39.  $\left\{-\frac{1}{2}\right\}$

40.  $\left\{\frac{\sqrt{5}}{5}\right\}$

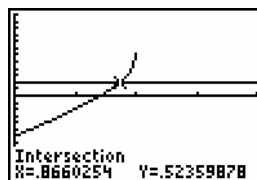
41.  $\{0\}$

42.  $\{0\}$

43.  $Y = \arcsin X - \arccos X - \frac{\pi}{6}$



44.  $Y_1 = \arcsin X - \arccos X$   $Y_2 = \frac{\pi}{6}$

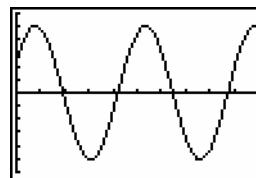


45.  $\{4.4622\}$

46.  $\{2.2824\}$

47. (a)  $A \approx .00506$ ;  $\phi \approx .484$ ;  
 $P = .00506 \sin(440\pi t + .484)$ .

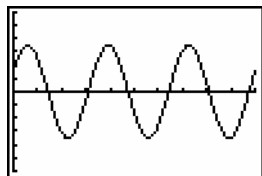
(b) For  $x = t$ ,  
 $P(t) = .00506 \sin(440\pi t + .484)$   
 $P_1(t) + P_2(t) = .0012 \sin(440\pi t + .052) + .004 \sin(440\pi t + .61)$



The two graphs are the same.

48. (a)  $\phi \approx .470$ ;  $f = 300$ ,  
 $P = .0035 \sin(600\pi t + .47)$ .

(b) For  $x = t$ ,  
 $P(t) = .0035 \sin(600\pi t + .47)$   
 $P_1(t) + P_2(t) = .0025 \sin\left(600\pi t + \frac{\pi}{7}\right) +$   
 $.001 \sin\left(600\pi t + \frac{\pi}{6}\right)$



The two graphs are the same.

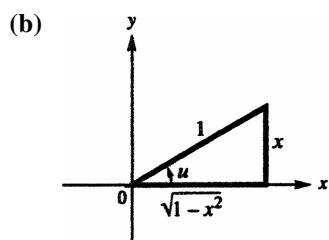
49. (a)  $\tan \alpha = \frac{x}{z}$ ;  $\tan \beta = \frac{x+y}{z}$

(b)  $\frac{x}{\tan \alpha} = \frac{x+y}{\tan \beta}$

(c)  $\alpha = \arctan\left(\frac{x \tan \beta}{x+y}\right)$

(d)  $\beta = \arctan\left(\frac{(x+y) \tan \alpha}{x}\right)$

50. (a)  $x = \sin u$ ,  $-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$



(c)  $\tan u = \frac{x\sqrt{1-x^2}}{1-x^2}$

(d)  $u = \arctan\left(\frac{x\sqrt{1-x^2}}{1-x^2}\right)$

51. (a)  $t = \frac{1}{2\pi f} \arcsin \frac{E}{E_{\max}}$

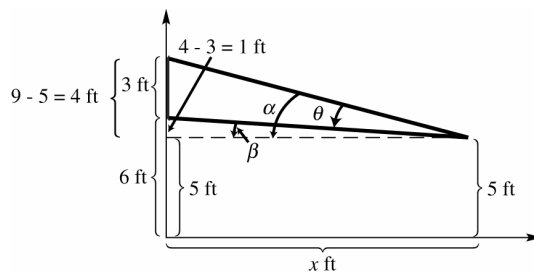
(b) .00068 sec

52. (a)  $\theta = \alpha - \beta$

Since  $\tan \alpha = \frac{4}{x} \Rightarrow \alpha = \tan^{-1}\left(\frac{4}{x}\right)$  and

$\tan \beta = \frac{1}{x} \Rightarrow \beta = \tan^{-1}\left(\frac{1}{x}\right)$ , we have

$\theta = \alpha - \beta \Rightarrow \theta = \tan^{-1}\left(\frac{4}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$



(b) (i) 4.26 ft or .94 ft

(ii) 6.64 ft or .60 ft

(c) (i) .54

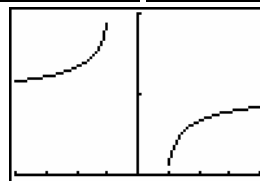
(ii) .61

53. (a)  $t = \frac{3}{4\pi} \arcsin 3y$

(b) .27 sec.

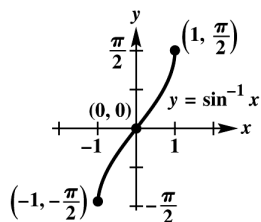
54.

Plot1	Plot2	Plot3	WINDOW
$\sqrt{1-(X^2)}$	$(\pi/2) - (\arcsin(X))$	$(\arcsin(X)) - (\pi/2)$	Xmin=-4
$1 - \sqrt{1-(X^2)}$	$(\pi/2) - \arcsin(X)$	$\arcsin(X) - (\pi/2)$	Xmax=4
$\sqrt{1-(X^2)}$	$(\pi/2) - \arcsin(X)$	$\arcsin(X) - (\pi/2)$	Xscl=1
$\sqrt{1-(X^2)}$	$(\pi/2) - \arcsin(X)$	$\arcsin(X) - (\pi/2)$	Ymin=0
$\sqrt{1-(X^2)}$	$(\pi/2) - \arcsin(X)$	$\arcsin(X) - (\pi/2)$	Ymax=3.1415926...
$\sqrt{1-(X^2)}$	$(\pi/2) - \arcsin(X)$	$\arcsin(X) - (\pi/2)$	Yscl=1.5707963...
$\sqrt{1-(X^2)}$	$(\pi/2) - \arcsin(X)$	$\arcsin(X) - (\pi/2)$	Xres=1

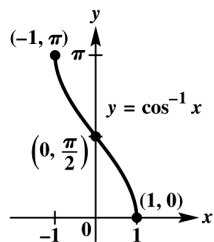


**Chapter 6: Review Exercises**

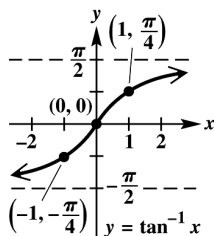
1.



$$[-1, 1]; \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



$$[-1, 1]; [0, \pi]$$



$$[-\infty, \infty]; \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

2. False. The range of the inverse tangent

function is  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , while the range of the inverse cotangent is  $(0, \pi)$ .

3. False.  $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ , not  $\frac{11\pi}{6}$ .

4. True

5.  $\frac{\pi}{4}$

6.  $\frac{2\pi}{3}$

7.  $-\frac{\pi}{3}$

8.  $-\frac{\pi}{2}$

9.  $\frac{3\pi}{4}$

10.  $\frac{\pi}{6}$

11.  $\frac{2\pi}{3}$

12.  $\frac{\pi}{3}$

13.  $\frac{3\pi}{4}$

14.  $60^\circ$

15.  $-60^\circ$

16.  $0^\circ$

17.  $\theta = 60.67924514^\circ$

18.  $\theta \approx -41.33444556^\circ$

19.  $\theta \approx 36.4895081^\circ$

20.  $\theta \approx 12.51631252^\circ$

21.  $\theta \approx 73.26220613^\circ$

22.  $\theta \approx 7.673567973^\circ$

23.  $-1$

24.  $-\frac{\sqrt{3}}{2}$

25.  $\frac{3\pi}{4}$

26.  $\pi$

27.  $\frac{\pi}{4}$

28.  $0$

29.  $\frac{\sqrt{7}}{4}$

30.  $\frac{\sqrt{10}}{10}$

31.  $\frac{\sqrt{3}}{2}$

32.  $\frac{9}{7}$

33.  $\frac{294 + 125\sqrt{6}}{92}$

34.  $\sqrt{1-u^2}$

35.  $\frac{1}{u}$

36.  $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

37.  $\{.4636, 3.6052\}$

38.  $\left\{.7297, \frac{\pi}{2}, 2.4119\right\}$

39.  $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

40.  $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

41.  $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

42.  $\{0 + 2n\pi, \text{ where } n \text{ is any integer}\}$  or  
 $\{2n\pi, \text{ where } n \text{ is any integer}\}$

43.  $\left\{\frac{\pi}{3} + 2n\pi, \pi + 2n\pi, \frac{5\pi}{3} + 2n\pi, \text{ where } n \text{ is any integer}\right\}$

44.  $\left\{\frac{\pi}{6} + 2n\pi, \frac{\pi}{3} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \text{ where } n \text{ is any integer}\right\}$  or  
 $\left\{\frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, \text{ where } n \text{ is any integer}\right\}$

45.  $\{270^\circ\}$

46.  $\{45^\circ, 153.4^\circ, 225^\circ, 333.4^\circ\}$

47.  $\{45^\circ, 90^\circ, 225^\circ, 270^\circ\}$

48.  $\{15^\circ, 75^\circ, 195^\circ, 255^\circ\}$

49.  $\{70.5^\circ, 180^\circ, 289.5^\circ\}$

50.  $\{53.5^\circ, 118.4^\circ, 233.5^\circ, 298.4^\circ\}$

51.  $x = \arcsin 2y$

52.  $x = 2 \arccos \frac{y}{3}$

53.  $x = \left(\frac{1}{3} \arctan 2y\right) - \frac{2}{3}$

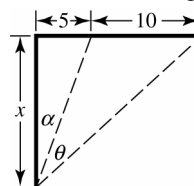
54.  $x = \arcsin\left(\frac{5y+3}{4}\right)$

55.  $\emptyset$

56.  $\left\{\frac{3\sqrt{5}}{7}\right\}$

57.  $\left\{-\frac{1}{2}\right\}$

58.  $t = \frac{50}{\pi} \arccos\left(\frac{d-550}{450}\right)$

59. (a) Let  $\alpha$  be the angle to the left of  $\theta$ .

Thus, we have

$$\tan(\alpha + \theta) = \frac{5+10}{x}$$

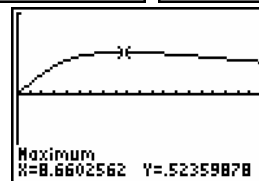
$$\alpha + \theta = \arctan\left(\frac{15}{x}\right)$$

$$\theta = \arctan\left(\frac{15}{x}\right) - \alpha$$

$$\theta = \arctan\left(\frac{15}{x}\right) - \arctan\left(\frac{5}{x}\right)$$

(b) The maximum occurs at approximately 8.66026 ft. There may be a discrepancy in the final digits.

Plot1	Plot2	Plot3	WINDOW
$\sqrt{15} \tan^{-1}(15/X) - t$			Xmin=0
$\arctan(5/X)$			Xmax=20
$\sqrt{Y_2=}$			Xscl=1
$\sqrt{Y_3=}$			Ymin=-1
$\sqrt{Y_4=}$			Ymax=1
$\sqrt{Y_5=}$			Yscl=1
$\sqrt{Y_6=}$			Xres=1



60.  $48.8^\circ$

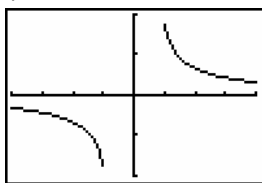
61. The light beam is completely underwater.

62. (a)  $42.2^\circ$

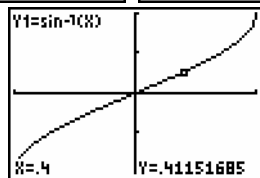
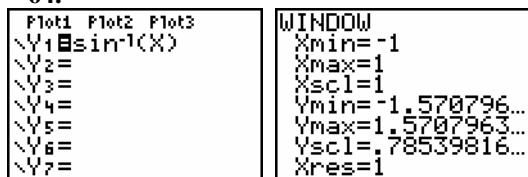
(b)  $90^\circ$

(c)  $48.0^\circ$

63.  $y = \csc^{-1} x$



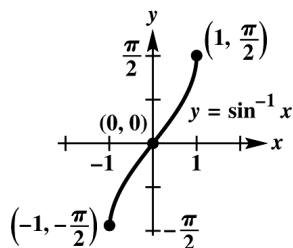
64.



(a),(b) In both cases,  $\sin^{-1}.4 \approx .41151685$

### Chapter 6 Test

1.



$[-1, 1]; \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. (a)  $\frac{2\pi}{3}$

(b)  $-\frac{\pi}{3}$

(c) 0

(d)  $\frac{2\pi}{3}$

3. (a)  $30^\circ$

(b)  $-45^\circ$

(c)  $135^\circ$

(d)  $-60^\circ$

4. (a)  $42.54^\circ$

(b)  $22.72^\circ$

(c)  $125.47^\circ$

5. (a)  $\frac{\sqrt{5}}{3}$

(b)  $\frac{4\sqrt{2}}{9}$

6. Since  $-1 \leq \sin \theta \leq 1$ , there is no value of  $\theta$  for which  $\sin \theta = 3$ . Thus,  $\sin^{-1} 3$  is not defined.

7.  $\arcsin\left(\sin \frac{5\pi}{6}\right) = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6} \neq \frac{5\pi}{6}$

8.  $\frac{u\sqrt{1-u^2}}{1-u^2}$

9.  $\{30^\circ, 330^\circ\}$

10.  $\{90^\circ, 270^\circ\}$

11.  $\{18.4^\circ, 135^\circ, 198.4^\circ, 315^\circ\}$

12.  $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

13.  $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}\right\}$

14.  $\{.3649, 1.2059, 3.5605, 4.3475\}$

15.  $\{90^\circ + 180^\circ n, \text{ where } n \text{ is any integer}\}$ .

16.  $\left\{\frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \text{ where } n \text{ is any integer}\right\}$

17. (a)  $x = \frac{1}{3} \arccos y$

(b)  $\left\{\frac{4}{5}\right\}$

18.  $\frac{5}{6} \text{ sec}, \frac{11}{6} \text{ sec}, \frac{17}{6} \text{ sec}$

### Chapter 6: Quantitative Reasoning

$20[9 \arctan(\sqrt{8}) - \sqrt{8}] \approx 165$  cubic ft