Chapter 7

Applications of Trigonometry and Vectors

Section 7.1: Oblique Triangles and the Law of Sines

Connections (page 307)

$$X = \frac{(a-h)x}{f\sec\theta - y\sin\theta}, Y = \frac{(a-h)y\cos\theta}{f\sec\theta - y\sin\theta}$$

1. house: $X_H \approx 1131.8$ ft ; $Y_H \approx 4390.2$ ft
forest fire: $X_F \approx 2277.5$ ft ; $Y_F \approx -2596.2$ ft

2. 7079.7 ft

Exercises

1. C

2. C, D

3. $\sqrt{3}$

4. $10\sqrt{2}$

5. $C = 95°$; $b \approx 13$ m; $a \approx 11$ m

6. $A = 99°$; $b \approx 34$ cm; $c \approx 21$ cm

7. $B = 37.3°$; $a \approx 38.5$ ft; $b \approx 51.0$ ft

8. $A = 37.2°$; $a \approx 178$ m ; $c \approx 244$ m

9. $C = 57.36°$; $b \approx 11.13$ ft ; $c \approx 11.55$ ft

10. $A = 36.54°$; $a \approx 28.10$ m ; $b \approx 44.17$ m

11. $B = 18.5°$ **;** $a \approx 239$ yd; $c \approx 230$ yd

12. $A = 49°40'$ **;** $b \approx 16.1$ cm; $c \approx 25.8$ cm

13. $A = 56°00'$; $AB \approx 361$ ft ; $BC \approx 308$ ft

14. $C = 91.9°$; $BC \approx 490$ ft; $AB \approx 847$ ft

15. $B \approx 110.0°$; $a \approx 27.01$ m; $c \approx 21.37$ m

16. $A = 65.60°$; $b \approx 1.942$ cm; $c \approx 2.727$ cm

17. $A = 34.72°$; $a \approx 3326$ ft ; $c \approx 5704$ ft

18. $C = 109.72°$; $a \approx 955.4$ yd ; $b \approx 2327$ yd

19. $C = 97°34'$ **;** $b \approx 283.2$ m; $c \approx 415.2$ m

20. $B = 67°45'$ **;** $b \approx 22.04$ mm; $c \approx 37.50$ mm

21. With three sides we have,
$\frac{\overbrace{a}}{\sin A} = \frac{\overbrace{b}}{\sin B} = \frac{\overbrace{c}}{\sin C}$. This does not provide enough information to solve the triangle. Whenever you choose two out of the three ratios to create a proportion, you are missing two pieces of information.

22. The triangle is not a right triangle.

23. This is not a valid statement. It is true that if you have the measures of two angles, the third can be found. However, if you do not have at least one side, the triangle cannot be uniquely determined. If we consider the congruence axiom involving two angles, ASA, an included side must be considered.

24. No

25. 118 m

26. 448 yd

27. 17.8 km

28. 1.93 mi

29. 10.4 in.

30. In triangle *MTL*, 324.9645
In right triangle *MTR*, 293.4 m

31. 111°

32. 12

33. First location: 5.1 mi
Second location: 7.2 mi

34. .49 mi

35. In either case the distance is approximately 419,000 km compared to the actual value of 406,000 km.

36. approximately 10,285 ft.

37. ≈ 6600 ft

38. ≈ 5100 ft

39. $\frac{\sqrt{3}}{2}$ sq unit

40. $\sqrt{3}$ sq unit **41.** $\frac{\sqrt{2}}{2}$ sq unit

42. 1 sq unit

43. 46.4 m^2

44. 732 ft^2

45. 356 cm^2

46. 163 km^2

47. 722.9 $in.^2$

48. 289.9 m^2

49. 65.94 cm^2

50. 84.41 m^2

51. 100 m^2

52. 373 m^2

53. $a = \sin A,\ b = \sin B,$ and $c = \sin C.$

54. Answers will vary.

55. $\frac{d \sin\alpha \sin\beta}{\sin(\beta - \alpha)} = x$

Section 7.2: The Ambiguous Case of the Law of Sines

1. A

2. D

3. **(a)** $4 < h < 5.$

(b) $h = 4$ or $h > 5$

(c) $h < 4$

4. **(a)** none

(b) $h \quad 5$

(c) $h \le 5$

5. 1

6. 0

7. 2

8. 1

9. 0

10. 2

11. 45°

12. 30°

13. $B_1 = 49.1°$; $B_2 = 130.9°$
$C_1 = 101.2°$;

14. $A_1 = 72.2°.$; $C_1 = 59.6°$
$A_2 = 107.8°,$; $C_2 = 24.0°$

15. $B_1 = 26°30'$; $A = 112°10'$.

16. $A = 37°50'$, $C = 93°20'$

17. no such triangle exists.

18. no such triangle exists.

19. $B \approx 27.19°$ and $C \approx 10.68°.$

20. $A = 45.40°$ and $C = 20.88°.$

21. $B_1 = 20.6°$; $C = 116.9°$; $c \approx 20.6$ ft

22. $A_1 = 25.5°$; $B = 102.2°$; $b \approx 73.9$ yd

23. no such triangle exists.

24. no such triangle exists.

25. $B_1 \approx 49°20'$; $C_1 = 92°00'$
$c_1 \approx 15.5$ km ; $c_2 \approx 2.88$ km

26. $A_1 \approx 55°20'$; $B_1 = 94°50'$
$b_1 \approx 10.4$ m ; $b_2 \approx 4.51$ m

27. $B_1 \approx 37.77°$; $C = 45.43°$; $c \approx 4.174$ ft

28. $B_1 \approx 30.39°$; $A = 60.91°$; $a \approx 98.25$ yd

29. $A_1 = 53.23°$; $C_1 = 87.09°$
$c_1 \approx 37.16$ m ; $A_2 = 126.77°$
$C_2 \approx 13.55°$; $c_2 = 8.719$ m

30. $C_1 = 59.71°.$; $B_1 = 69.09°$;
$b_1 \approx 8640$ cm ; $B_2 = 8.51°$; $b_2 \approx 1369$ cm

31. **1;** 90° ; a right triangle.

32.–34. Answers will vary.

35. 664 m

36. 87.3 ft

37. 218 ft

38. 75° ; 274 mi

39. Prove that $\dfrac{a+b}{b}=\dfrac{\sin A+\sin B}{\sin B}$.
Start with the law of sines.
$$\frac{a}{\sin A}=\frac{b}{\sin B}\Rightarrow a=\frac{b\sin A}{\sin B}$$
Substitute for a in the expression $\dfrac{a+b}{b}$.
$$\frac{a+b}{b}=\frac{\frac{b\sin A}{\sin B}+b}{b}=\frac{\frac{b\sin A}{\sin B}+b}{b}\cdot\frac{\sin B}{\sin B}$$
$$=\frac{b\sin A+b\sin B}{b\sin B}=\frac{b\left(\sin A+\sin B\right)}{\sin B}$$
$$=\frac{\sin A+\sin B}{\sin B}$$

40. Prove that $\dfrac{a-b}{a+b}=\dfrac{\sin A-\sin B}{\sin A+\sin B}$.
Start with the law of sines.
$$\frac{a}{\sin A}=\frac{b}{\sin B}\Rightarrow a=\frac{b\sin A}{\sin B}$$
Substitute for a in the expression $\dfrac{a-b}{a+b}$.
$$\frac{a-b}{a+b}=\frac{\frac{b\sin A}{\sin B}-b}{\frac{b\sin A}{\sin B}+b}=\frac{\frac{b\sin A}{\sin B}-b}{\frac{b\sin A}{\sin B}+b}\cdot\frac{\sin B}{\sin B}$$
$$=\frac{b\sin A-b\sin B}{b\sin A+b\sin B}=\frac{b\left(\sin A-\sin B\right)}{b\left(\sin A+\sin B\right)}$$
$$=\frac{\sin A-\sin B}{\sin A+\sin B}$$

Section 7.3: The Law of Cosines

Connections (page 324)

1. Answers will vary.
2. Answers will vary.

Exercises

1. **(a)** SAS
   (b) law of cosines
2. A, C, and c
   (a) SAA
   (b) law of sines
3. **(a)** SSA
   (b) law of sines
4. **(a)** SSS
   (b) law of cosines
5. **(a)** ASA
   (b) law of sines
6. **(a)** SSA
   (b) law of sines
7. **(a)** ASA
   (b) law of sines
8. **(a)** SAS
   (b) law of cosines
9. 5
10. 7
11. 120°
12. 30°
13. $a\approx 7.0$; $C\approx 21.4°$; $B=37.6°$
14. $a\approx 5.4$; $B\approx 40.7°$; $C=78.3°$
15. $B\approx 53.1°$; $C=53.1°$; $A=73.7°$ The angles may not sum to 180° due to rounding.
16. $B\approx 108.2°$; $A\approx 22.3°$; $C=49.5°$
17. $b\approx 88.18$; $A\approx 56.7°$; $C=68.3°$
18. $C\approx 95.7°$; $A\approx 33.6°$; $B=50.7°$
19. $a\approx 2.60$ yd; $B\approx 45.1°$; $C=93.5°$
20. $c\approx 2.83$ in.; $A\approx 44.9°$; $B=106.8°$
21. $c\approx 6.46$ m; $A\approx 53.1°$; $B=81.3°$
22. $a\approx 43.7$ km; $B\approx 53.2°$; $C=59.5°$
23. $A\approx 82°$; $B\approx 37°$; $C=61°$
24. $C\approx 98°$; $A\approx 29°$; $B=53°$
25. $C\approx 102°10'$; $B\approx 35°50'$; $A=42°00'$
26. $C\approx 107°20'$; $B\approx 39°00'$; $A=33°40'$
27. $C\approx 84°30'$; $B\approx 44°40'$; $A=50°50'$
28. $B\approx 85°10'$; $C\approx 44°50'$; $A=50°00'$
29. $a\approx 156$ cm; $C=34°30'$; $B=64°50'$
30. $c\approx 348$ ft; $B=43°30'$; $A=63°50'$
31. $b\approx 9.529$ in.; $C\approx 40.61°$; $A=64.59°$
32. $c\approx 4.276$ mi; $A\approx 48.77°$; $B=71.53°$
33. $a\approx 15.7$ m; $B\approx 21.6°$; $C=45.6°$
34. $b\approx 34.1$ cm; $A\approx 5.2°$; $C=6.6°$
35. $C\approx 94°$; $A\approx 30°$; $B=56°$
36. $C\approx 125°$; $A\approx 24°$; $B=31°$

37. The value of cos θ will be greater than 1; your calculator will give you an error message (or a nonreal, complex number) when using the inverse cosine function.

38. Answers will vary.

39. 257 m.

40. 5.2 cm and 8.8 cm.

41. 281 km

42. 745 mi

43. 10.8 miles.

44. 1450 ft.

45. $\theta \approx 40°$

46. $AB \approx 18$ ft

47. $A \approx 36°$; $B \approx 26°$

48. about 5500 meters long

49. second base is 66.8 ft and the distance to both first and third base is 63.7 ft.

50. 438.14 feet.

51. $v \approx 39.2$ km

52. 1473 m apart

53. 350°

54. approximately 180 mi.

55. 47.5 feet

56. 5.9 mi

57. $A \approx 163.5°$

58. 5.99 km

59. 22 ft

60. 3921 m

61. $\theta \approx 16.26°$

62. $\theta \approx 14.25°$

63. $24\sqrt{3}$ sq units

64. $15\sqrt{3}$ sq units

65. 78 m^2 (rounded to two significant digits)

66. 310 $in.^2$ (rounded to two significant digits)

67. 12,600 cm^2

68. 228 yd^2

69. 3650 ft^2 (rounded to three significant digits)

70. 83.01 $in.^2$

71. 25.24983 mi.

72. 71.99670°

73. Since the perimeter and area both equal 36 feet, the triangle is a *perfect triangle.*

74. **(a)** $A = 66,$ which is an integer

(b) $A = 84,$ which is an integer

(c) $A = 42,$ which is an integer

(d) $A = 36,$ which is an integer

75. 390,000 mi^2.

76. 33 cans

77. **(a)** $C_1 = 87.8°.$; B_2 92.2° both appear possible.

(b) 92.2°

(c) With the law of cosines, we are required to find the inverse cosine of a negative number; therefore; we know angle *C* is greater than 90°.

78. Using the law of cosines, we have

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow$$

$$\cos B = \frac{6^2 + 5^2 - 4^2}{2(6)(5)} = \frac{36 + 25 - 16}{60} = \frac{3}{4}$$

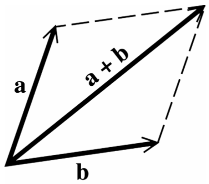
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow$$

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2(4)(5)} = \frac{16 + 25 - 36}{40} = \frac{1}{8}$$

Since $2\cos^2 B - 1 = 2\left(\frac{3}{4}\right)^2 - 1 = 2\left(\frac{9}{16}\right) - \frac{16}{16}$

$= \frac{2}{16} = \frac{1}{8} = \cos A,$ A is twice the size of B.

9.



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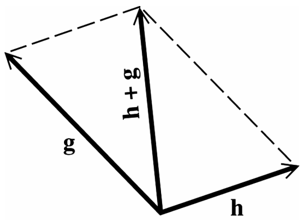

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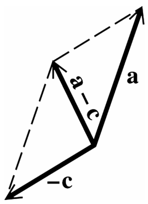

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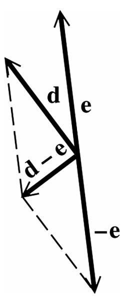

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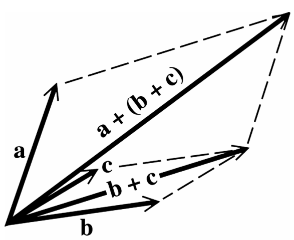

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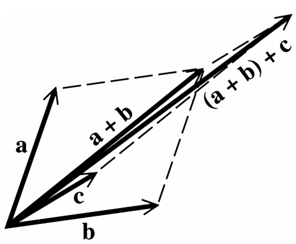

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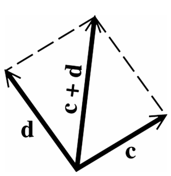

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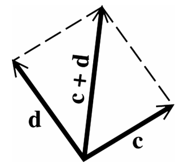

17. Yes, vector addition is associative.

18. Yes, vector addition is commutative.

19. **(a)** $\langle -4, 16\rangle$

(b) $\langle -12, 0\rangle$

(c) $\langle 8, -8\rangle$

20. **(a)** $\langle -4, -8\rangle$

(b) $\langle 12, 0\rangle$

(c) $\langle -4, 4\rangle$

21. **(a)** $\langle 8, 0\rangle$

(b) $\langle 0, 16\rangle$

(c) $\langle -4, -8\rangle$

22. **(a)** $\langle 4, 0\rangle$

(b) $\langle -12, -8\rangle$

(c) $\langle 4, 4\rangle$

23. **(a)** $\langle 0, 12\rangle$

(b) $\langle -16, -4\rangle$

(c) $\langle 8, -4\rangle$

24. **(a)** $\langle 4, 4\rangle$

(b) $\langle 12, -12\rangle$

(c) $\langle -8, 4\rangle$

25. **(a)** $4\mathbf{i}$

(b) $7\mathbf{i} + 3\mathbf{j}$

(c) $-5\mathbf{i} + \mathbf{j}$

26. **(a)** $-2\mathbf{i} + 4\mathbf{j}$

(b) $\mathbf{i} + \mathbf{j}$

(c) $4\mathbf{i} - 7\mathbf{j}$

54. $|\mathbf{v}| \approx 158.0$ lb

55. $|\mathbf{v}| \approx 24.4$ lb

56. $|\mathbf{v}| \approx 1286.0$ lb

57. $\boldsymbol{u} + \boldsymbol{v} = \langle a+c, b+d \rangle$.

58. $z_1 + z_2 = (a+bi)+(c+di) = (a+c)+(b+d)i$
Additional answers will vary.

59. $\mathbf{u} + \mathbf{v} = \langle 2, 8 \rangle$

60. $\mathbf{u} - \mathbf{v} = \langle -6, 2 \rangle$

61. $-4\mathbf{u} = \langle 8, -20 \rangle$

62. $-5\mathbf{v} = \langle -20, -15 \rangle$

63. $3\mathbf{u} - 6\mathbf{v} = \langle -30, -3 \rangle$

64. $-2\mathbf{u} + 4\mathbf{v} = \langle 20, 2 \rangle$

65. $\mathbf{u} + \mathbf{v} - 3\mathbf{u} = \langle 8, -7 \rangle$

66. $2\mathbf{u} + \mathbf{v} - 6\mathbf{v} = \langle -24, -5 \rangle$

67. $\langle -5, 8 \rangle = -5\mathbf{i} + 8\mathbf{j}$

68. $\langle 6, -3 \rangle = 6\mathbf{i} - 3\mathbf{j}$

69. $\langle 2, 0 \rangle = 2\mathbf{i} + 0\mathbf{j} = 2\mathbf{i}$

70. $\langle 0, -4 \rangle = 0\mathbf{i} - 4\mathbf{j} = -4\mathbf{j}$

71. 7

72. −61

73. −3

74. 1

75. 20

76. −4

77. $\theta = 135°$

78. $\theta \approx 36.87°$

79. $= 90°$

80. $\theta = 45°$

81. $\theta \approx 36.87°$

82. $\theta \approx 78.93°$

83. −6

84. −24

85. −24

86. −6

87. the vectors are orthogonal.

88. the vectors are not orthogonal.

89. the vectors are not orthogonal

90. the vectors are orthogonal.

91. the vectors are not orthogonal.

92. the vectors are not orthogonal

93. Draw a line parallel to the *x*-axis and the vector **u + v** (shown as a dashed line)
Since $\theta_1 = 110°$, its supplementary angle is 70°. Further, since $\theta_2 = 260°$, the angle α is $260° - 180° = 80°$. Then the angle *CBA* becomes 180 – (80 + 70) = 180 – 150 = 30°.

Magnitude: ≈ 9.5208
The direction angle is $119.0647°$

94. $\langle a, b \rangle \approx \langle -4.1042, 11.2763 \rangle$.

95. $\langle c, d \rangle \approx \langle -.5209, -2.9544 \rangle$.

31. **(a)** $|\boldsymbol{R}| = \sqrt{5} \approx 2.2$ and $|\boldsymbol{A}| = \sqrt{1.25} \approx 1.1$
About 2.2 in. of rain fell. The area of the opening of the rain gauge is about 1.1 in.^{2}.

(b) $V = 1.5$; The volume of rain was 1.5 in.^{3}.

(c) **R** and **A** should be parallel and point in opposite directions.

32. $\boldsymbol{a} = \langle a_1, a_2 \rangle, \boldsymbol{b} = \langle b_1, b_2 \rangle$, and
$\boldsymbol{a} - \boldsymbol{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$

$$|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2|\mathbf{a}||\mathbf{b}|\cos\theta$$
$$\mathrm{a} \cdot \mathrm{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

Summary Exercises on Applications of Trigonometry and Vectors

1. The length of the two wires are about 29 ft and 38 ft.

2. $TM \approx 43$ ft

3. 38.3 cm

4. 5856 m apart.

5. $x \approx 15.8$ ft per sec ; $71.6°$

6. 42 lb

7. 7200 ft above the ground

8. **(a)** The speed of the wind is 10 mph

(b) This represents a 30 mph wind in the direction of **v**.

(c) **u** represents a southeast wind of 11.3 mph

9. Since $-1 \le \sin A \le 1$, the triangle cannot exist.

10. Other angles can be $36°10'$; third side 40.5, or other angles can be 143º50′, 8º00′ and third side 6.25.(Lengths in yards.)

Chapter 7: Review Exercises

1. 63.7 m

2. $B \approx 25.0°$.

3. $B = 41.7°$.

4. 70.9 m

5. $54°20'$ or $125°40'$

6. $A = 49°30'$.

7. No; If you are given two angles of a triangle, then the third angle is known since the sum of the measures of the three angles is $180°$. Since you are also given one side, there will only be one triangle that will satisfy the conditions.

8. No; the sum of a and b do not exceed c.

9. $a = 10$, $B = 30°$

(a) $b = 10 \sin 30° = 5$. Also, any value of b greater than or equal to 10 would yield a unique value for A.

(b) Any value of b between 5 and 10, would yield two possible values for A.

(c) If b is less than 5, then no value for A is possible.

10. $A = 140°$, $a = 5$, and $b = 7$
With these conditions, we can try to solve the triangle with the law of sines.

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \frac{\sin B}{7} = \frac{\sin 140°}{5} \Rightarrow$$
$$\sin B = \frac{7 \sin 140°}{5} \approx .89990265 \Rightarrow B \approx 64°$$

Since $A + B = 140° + 64° = 204° > 180°$, no such triangle exists.

11. $A \approx 19.87°$ or $19°\ 52'$.

12. 173 ft

13. 55.5 m

14. $B \approx 26.5°$ or $26°\ 30'$.

15. 19 cm

16. $47°$

17. $B = 17.3°$; $C = 137.5°$; $c \approx 11.0$ yd

18. $B_1 = 74.6°$; $C_1 = 43.7°$; $c_1 \approx 61.9$ m ; $C_2 = 12.9°$; $c_2 \approx 20.0$ m

19. $c \approx 18.65$ cm ; $B \approx 45°50'$; $A = 91°\ 40'$

20. $B \approx 73.9°$; $A \approx 47.7°$; $C = 58.4°$

21. $153{,}600$ m^2

22. 20.3 ft^2

23. .234 km^2

24. 680 m^2

25. 58.6 feet

26. 11 feet long

27. 13 meters tall

12. 2.7 miles off the ground

13. $\langle -346, 451 \rangle$.

14. 1.91 mi

15. 14 m

16. 30 lb

Chapter 7: Quantitative Reasoning

1. We can use the area formula $A = \frac{1}{2} rR \sin B$ for this triangle. By the law of sines, we have

$$\frac{r}{\sin A} = \frac{R}{\sin C} \Rightarrow r = \frac{R \sin A}{\sin C}$$

Since

$\sin C = \sin\left[180° - (A+B)\right] = \sin(A+B)$, we have $r = \frac{R\sin A}{\sin C} \Rightarrow r = \frac{R \sin A}{\sin(A+B)}$

By substituting into our area formula, we have

$$A = \frac{1}{2} rR\sin B \Rightarrow$$

$$A = \frac{1}{2}\left[\frac{R\sin A}{\sin(A+B)}\right] R\sin B \Rightarrow$$

$$A = \frac{1}{2} \cdot \frac{\sin A \sin B}{\sin(A+B)} R^2$$

Since there are a total of 10 stars, the total area covered by the stars is

$$A = 10\left[\frac{1}{2} \cdot \frac{\sin A \sin B}{\sin(A+B)} R^2\right] = \left[5\frac{\sin A \sin B}{\sin(A+B)}\right] R^2$$

2. $1.12257R^2$

3. **(a)** 8.77 in.^{2}

(b) 5.32 in.^{2}

(c) red