There is an expanding market for high-speed trains as more countries, such as China, turn to bullet trains. In April 2007, a French TGV, short for “train à grande vitesse” or “very fast train,” broke the previous world speed record on rails. (It should be noted that the current record for overall train speed is held by a magnetically levitated train called the Maglev, from Japan. This train hovers above the rails.)

The bar graph below shows a history of train speed records. In Section 2.4, Exercise 49, you will have the opportunity to calculate the speeds of the Maglev and the TGV.

Much of mathematics relates to deciding which statements are true and which are false. For example, the statement \( x + 7 = 15 \) is an equation stating that the sum \( x + 7 \) has the same value as 15. Is this statement true or false? It is false for some values of \( x \) and true for just one value of \( x \), namely 8. Our purpose in this chapter is to learn ways of deciding which values make an equation or an inequality true.

### Train Speed Records

<table>
<thead>
<tr>
<th>Year</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1829</td>
<td>34</td>
</tr>
<tr>
<td>1890</td>
<td>89.4</td>
</tr>
<tr>
<td>1891</td>
<td>96</td>
</tr>
<tr>
<td>1903</td>
<td>130</td>
</tr>
<tr>
<td>1955</td>
<td>285</td>
</tr>
<tr>
<td>1981</td>
<td>236</td>
</tr>
<tr>
<td>1990</td>
<td>320.2</td>
</tr>
<tr>
<td>2007</td>
<td></td>
</tr>
<tr>
<td>2003*</td>
<td></td>
</tr>
</tbody>
</table>

* The Japanese Maglev is often not included in railway records since it hovers above the rails.

### Source
International Herald Tribune
As we explore in this section, an expression such as $3x + 2x$ is not as simple as possible, because—even without replacing $x$ by a value—we can perform the indicated addition.

**OBJECTIVE 1** Identifying terms, like terms, and unlike terms. Before we practice simplifying expressions, some new language of algebra is presented. A **term** is a number or the product of a number and variables raised to powers.

**Terms**

\[-y, \ 2x^3, \ -5, \ 3xz^2, \ \frac{2}{y}, \ 0.8z\]

The **numerical coefficient** (sometimes also simply called the **coefficient**) of a term is the numerical factor. The numerical coefficient of $3x$ is $3$. Recall that $3x$ means $3 \cdot x$.

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x$</td>
<td>$3$</td>
</tr>
<tr>
<td>$\frac{y^3}{5}$</td>
<td>$\frac{1}{5}$ since $\frac{y^3}{5}$ means $\frac{1}{5} \cdot y^3$</td>
</tr>
<tr>
<td>$0.7ab^2c^5$</td>
<td>$0.7$</td>
</tr>
<tr>
<td>$z$</td>
<td>$1$</td>
</tr>
<tr>
<td>$-y$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$-5$</td>
<td>$-5$</td>
</tr>
</tbody>
</table>

**Helpful Hint**

The term $-y$ means $-1y$ and thus has a numerical coefficient of $-1$. The term $z$ means $1z$ and thus has a numerical coefficient of $1$.

**EXAMPLE 1** Identify the numerical coefficient in each term.

**a.** $-3y$    **b.** $22z^4$    **c.** $y$    **d.** $-x$    **e.** $\frac{x}{7}$

**Solution**

a. The numerical coefficient of $-3y$ is $-3$.
b. The numerical coefficient of $22z^4$ is $22$.
c. The numerical coefficient of $y$ is $1$, since $y$ is $1y$.
d. The numerical coefficient of $-x$ is $-1$, since $-x$ is $-1x$.
e. The numerical coefficient of $\frac{x}{7}$ is $\frac{1}{7}$, since $\frac{x}{7}$ means $\frac{1}{7} \cdot x$.

**PRACTICE 1** Identify the numerical coefficients in each term.

**a.** $t$    **b.** $-7x$    **c.** $-\frac{16}{5}$    **d.** $43x^4$    **e.** $-b$

Terms with the same variables raised to exactly the same powers are called **like terms**. Terms that aren’t like terms are called **unlike terms**.
OBJECTIVE 2 Combining like terms. An algebraic expression containing the sum or difference of like terms can be simplified by applying the distributive property. For example, by the distributive property, we rewrite the sum of the like terms as 

\[3x + 2x = (3 + 2)x = 5x\]

Also, 

\[-y^2 + 5y^2 = (-1 + 5)y^2 = 4y^2\]

Simplifying the sum or difference of like terms is called \textbf{combining like terms}.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{Like Terms} & \textbf{Unlike Terms} \\
\hline
3x, 2x & 5x, 5x^2 \quad \text{Why? Same variable x, but different powers x and x}^2 \\
-6x^2y, 2x^2y, 4x^2y & 7y, 3z, 8x^2 \quad \text{Why? Different variables} \\
2ab^2c^3, ac^3b^2 & 6abc^2, 6ab^2 \quad \text{Why? Different variables and different powers} \\
\hline
\end{tabular}
\end{table}

\textbf{EXAMPLE 2} Determine whether the terms are like or unlike.

\begin{enumerate}
\item \(2x, 3x^2\)
\item \(4x^2y, x^2y, -2x^2y\)
\item \(-2yz, -3y\)
\item \(-x^4, x^4\)
\end{enumerate}

\textit{Solution}

\begin{enumerate}
\item Unlike terms, since the exponents on \(x\) are not the same.
\item Like terms, since each variable and its exponent match.
\item Like terms, since \(zy = yz\) by the commutative property.
\item Like terms.
\end{enumerate}

\textbf{EXAMPLE 3} Simplify each expression by combining like terms.

\begin{enumerate}
\item \(7x - 3x\)
\item \(10y^2 + y^2\)
\item \(8x^2 + 2x - 3x\)
\end{enumerate}

\textit{Solution}

\begin{enumerate}
\item \(7x - 3x = (7 - 3)x = 4x\)
\item \(10y^2 + y^2 = 10y^2 + 1y^2 = (10 + 1)y^2 = 11y^2\)
\item \(8x^2 + 2x - 3x = 8x^2 + (2 - 3)x = 8x^2 - x\)
\end{enumerate}

\textbf{PRACTICE 3} Simplify each expression by combining like terms.

\begin{enumerate}
\item \(4x^2 + 3x^2\)
\item \(-3y + y\)
\item \(5x - 3x^2 + 8x^2\)
\end{enumerate}
EXAMPLE 4  Simplify each expression by combining like terms.

a. $2x + 3x + 5 + 2$

b. $-5a - 3 + a + 2$

c. $4y - 3y^2$

d. $2.3x + 5x - 6$

e. $\frac{1}{2}b + b$

Solution  Use the distributive property to combine like terms.

a. $2x + 3x + 5 + 2 = (2 + 3)x + (5 + 2) = 5x + 7$

b. $-5a - 3 + a + 2 = -5a + 1a + (-3 + 2) = (-5 + 1)a + (-3 + 2) = -4a - 1$

c. $4y - 3y^2$  These two terms cannot be combined because they are unlike terms.

d. $2.3x + 5x - 6 = (2.3 + 5)x - 6 = 7.3x - 6$

e. $\frac{1}{2}b + b = \frac{1}{2}b + 1b = \left(-\frac{1}{2} + 1\right)b = \frac{1}{2}b$

PRACTICE  Use the distributive property to combine like terms.

a. $3y + 8y - 7 + 2$

b. $6x - 3 - x - 3$

c. $\frac{3}{4}t - t$

d. $9y + 3.2y + 10 + 3$

e. $5z - 3z^4$

The examples above suggest the following:

Combining Like Terms

To **combine like terms**, add the numerical coefficients and multiply the result by the common variable factors.

OBJECTIVE 3  Using the distributive property. Simplifying expressions makes frequent use of the distributive property to also remove parentheses.

EXAMPLE 5  Find each product by using the distributive property to remove parentheses.

a. $5(x + 2)$

b. $-2(y + 0.3z - 1)$

c. $-(x + y - 2z + 6)$

Solution  

a. $5(x + 2) = 5 \cdot x + 5 \cdot 2 = 5x + 10$  Apply the distributive property.  Multiply.

b. $-2(y + 0.3z - 1) = -2(y) + (-2)(0.3z) + (-2)(-1)$  Apply the distributive property.  Multiply.

$= -2y - 0.6z + 2$

c. $-(x + y - 2z + 6) = -1(x + y - 2z + 6)$  Distribute $-1$ over each term.

$= -1(x) - 1(y) - 1(-2z) - 1(6)$

$= -x - y + 2z - 6$

PRACTICE  Find each product by using the distributive property to remove parentheses.

a. $3(2x - 7)$

b. $-5(3x - 4z - 5)$

c. $-(2x - y + z - 2)$
Section 2.1  Simplifying Algebraic Expressions

Helpful Hint
If a “−” sign precedes parentheses, the sign of each term inside the parentheses is changed when the distributive property is applied to remove parentheses.

Examples:
\[-(2x + 1) = -2x - 1 \quad -(5x + y - z) = 5x - y + z\]
\[-(x - 2y) = -x + 2y \quad -(3x - 4y - 1) = 3x + 4y + 1\]

When simplifying an expression containing parentheses, we often use the distributive property in both directions—first to remove parentheses and then again to combine any like terms.

Example 6  Simplify the following expressions.

a. \(3(2x - 5) + 1\)  
b. \(-2(4x + 7) - (3x - 1)\)  
c. \(9 - 3(4x + 10)\)

Solution

a. \(3(2x - 5) + 1 = 6x - 15 + 1\)  
Apply the distributive property.  
\(= 6x - 14\)  
Combine like terms.

b. \(-2(4x + 7) - (3x - 1) = -8x - 14 - 3x + 1\)  
Apply the distributive property.  
\(= -11x - 13\)  
Combine like terms.

c. \(9 - 3(4x + 10) = 9 - 12x - 30\)  
Apply the distributive property.  
\(= -21 - 12x\)  
Combine like terms.

Practice 6  Simplify the following expressions.

a. \(4(9x + 1) + 6\)  
b. \(-7(2x - 1) - (6 - 3x)\)  
c. \(8 - 5(6x + 5)\)

Objective 4  Writing word phrases as algebraic expressions. Next, we practice writing word phrases as algebraic expressions.

Example 7  Write the phrase below as an algebraic expression. Then simplify if possible.

“Subtract 4x − 2 from 2x − 3.”

Solution  “Subtract 4x − 2 from 2x − 3” translates to \((2x - 3) - (4x - 2)\). Next, simplify the algebraic expression.

\[(2x - 3) - (4x - 2) = 2x - 3 - 4x + 2\]  
Apply the distributive property.  
\[= -2x - 1\]  
Combine like terms.

Practice 7  Write the phrase below as an algebraic expression. Then simplify if possible.

“Subtract 7x − 1 from 2x + 3.”
EXAMPLE 8 Write the following phrases as algebraic expressions and simplify if possible. Let \( x \) represent the unknown number.

a. Twice a number, added to 6
b. The difference of a number and 4, divided by 7
c. Five added to 3 times the sum of a number and 1
d. The sum of twice a number, 3 times the number, and 5 times the number

Solution

a. In words: twice a number added to 6
   Translate: \( 2x + 6 \)

b. In words: the difference of a number and 4 divided by 7
   Translate: \( \frac{x - 4}{7} \)

c. In words: five added to 3 times the sum of a number and 1
   Translate: \( 5 + 3(x + 1) = 5 + 3x + 3 \) Use the distributive property.
   Combine like terms.
   \( = 8 + 3x \)

Now let’s simplify.

\[
2x + 3x + 5x = 10x \quad \text{Combine like terms.}
\]
**Section 2.1 Simplifying Algebraic Expressions**

**VOCABULARY & READINESS CHECK**

Use the choices below to fill in each blank. Some choices may be used more than once.

- like
- numerical coefficient
- term
- distributive
- unlike
- combine like terms
- expression

1. $23y^2 + 10y - 6$ is called a(n) ____________ while $23y^2$, $10y$, and $-6$ are each called a(n) ____________.
2. To simplify $x + 4x$, we ____________.
3. The term $y$ has an understood ____________ of 1.
4. The terms $y$ and $y$ are terms and the terms $y$ and $y$ are terms.
5. For the term $\frac{1}{2}xy^2$, the number $\frac{1}{2}$ is the ____________.
6. $5(3x - y)$ equals $15x - 5y$ by the ____________ property.

**Fill in the blank with the numerical coefficient of each term. See Example 1.**

7. $-7y$ ____________
8. $3x$ ____________
9. $x$ ____________
10. $-y$ ____________
11. $-\frac{5}{3}$ ____________
12. $-\frac{2}{3}$ ____________

**Indicate whether the following lists of terms are like or unlike. See Example 2.**

13. $5y$, $-y$ ____________
14. $-2x^3y$, $6xy$ ____________
15. $2z$, $3z^2$ ____________
16. $b^2a$, $-\frac{7}{8}ab^2$ ____________

**EXERCISE SET**

Simplify each expression by combining any like terms. See Examples 3 and 4.

1. $7y + 8y$
2. $3x + 2x$
3. $8w - w + 6w$
4. $c - 7c + 2c$
5. $3h - 5 - 10b - 4$
6. $6g + 5 - 3g - 7$
7. $m - 4m + 2m - 6$
8. $a + 3a - 2 - 7a$
9. $5g - 3 - 5 - 5g$
10. $8p + 4 - 8p - 15$
11. $6.2x - 4 + x - 1.2$
12. $7.9y - 0.7 - y + 0.2$
13. $6x - 5x + x - 3 + 2x$
14. $8h + 13h - 6 + 7h - h$
15. $7x^2 + 8x^2 - 10x^2$
16. $8x^3 + x^3 - 11x^3$
17. $6x + 0.5 - 4.3x - 0.4x + 3$
18. $0.4y - 6.7 + y - 0.3 - 2.6y$
19. In your own words, explain how to combine like terms.
20. Do like terms contain the same numerical coefficients? Explain your answer.

Simplify each expression. First use the distributive property to remove any parentheses. See Examples 5 and 6.

21. $5(y - 4)$
22. $7(r - 3)$
23. $-2(x + 2)$
24. $-4(y + 6)$
25. $7(d - 3) + 10$
26. $9(z + 7) - 15$
27. $-5(2x - 3y + 6)$
28. $-2(4x - 3z - 1)$
29. $-(3x - 2y + 1)$
30. $-(y + 5z - 7)$
31. $5(x + 2) - (3x - 4)$
32. $4(2x - 3) - 2(x + 1)$

Write each of the following as an algebraic expression. Simplify if possible. See Example 7.

33. Add $6x + 7$ to $4x - 10$.
34. Add $3y - 5$ to $y + 16$.
35. Subtract $7x + 1$ from $3x - 8$.
36. Subtract $4x - 7$ from $12 + x$.
37. Subtract $5m - 6$ from $m - 9$.
38. Subtract $m - 3$ from $2m - 6$. 
MIXED PRACTICE

Simplify each expression. See Examples 3 through 7.

39. $2k - k - 6$
40. $7c - 8 - c$
41. $-9x + 4x + 18 - 10x$
42. $5y - 14 + 7y - 20y$
43. $-4(3y - 4) + 12y$
44. $-3(2x + 5) - 6x$
45. $3(2x - 5) - 5(x - 4)$
46. $2(6x - 1) - (x - 7)$
47. $-2(3x - 4) + 7x - 6$
48. $8y - 2 - (y + 4)$
49. $5k - (3k - 10)$
50. $-11c - (4 - 2c)$
51. Subtract $6x - 1$ from $3x + 4$
52. Subtract $4 + 3y$ from $8 - 5y$
53. $3.4m - 4 - 3.4m - 7$
54. $2.8u - 0.9 - 0.5 - 2.8w$
55. $\frac{1}{3}(7y - 1) + \frac{1}{6}(4y + 7)$
56. $\frac{1}{5}(9y + 2) + \frac{1}{10}(2y - 1)$
57. $2 + 4(6x - 6)$
58. $8 + 4(3x - 4)$
59. $0.5(m + 2) + 0.4m$
60. $0.2(k + 8) - 0.1k$
61. $10 - 3(2x + 3y)$
62. $14 - 11(5m + 3n)$
63. $6(3x - 6) - 2(x + 1) - 17x$
64. $7(2x + 5) - 4(x + 2) - 20x$
65. $\frac{1}{2}(12x - 4) - (x + 5)$
66. $\frac{1}{3}(9x - 6) - (x - 2)$

Write each phrase as an algebraic expression and simplify if possible. Let $x$ represent the unknown number. See Examples 7 and 8.

67. Twice a number, decreased by four
68. The difference of a number and two, divided by five
69. Seven added to double a number
70. Eight more than triple a number
71. Three-fourths of a number, increased by twelve
72. Eleven, increased by two-thirds of a number
73. The sum of 5 times a number and $-2$, added to 7 times the number
74. The sum of 3 times a number and 10, subtracted from 9 times the number
75. Eight times the sum of a number and six
76. Six times the difference of a number and five
77. Double a number, minus the sum of the number and ten
78. Half a number, minus the product of the number and eight
79. Seven, multiplied by the quotient of a number and six
80. The product of a number and ten, less twenty
81. The sum of 2, three times a number, $-9$, and four times the number
82. The sum of twice a number, $-1$, five times the number, and $-12$

REVIEW AND PREVIEW

Evaluate the following expressions for the given values. See Section 1.7.

83. If $x = -1$ and $y = 3$, find $y - x^2$.
84. If $g = 0$ and $h = -4$, find $gh - h^2$.
85. If $a = 2$ and $b = -5$, find $a - b^2$.
86. If $x = -3$, find $x^3 - x^2 + 4$.
87. If $y = -5$ and $z = 0$, find $yz - y^2$.
88. If $x = -2$, find $x^3 - x^2 - x$.

CONCEPT EXTENSIONS

89. Recall that the perimeter of a figure is the total distance around the figure. Given the following rectangle, express the perimeter as an algebraic expression containing the variable $x$.

![Rectangle diagram]

90. Given the following triangle, express its perimeter as an algebraic expression containing the variable $x$.

![Triangle diagram]

Given the following two rules, determine whether each scale in Exercises 91 through 94 is balanced or not.

1. 1 cone balances 1 cube
2. 1 cylinder balances 2 cubes
Write each algebraic expression described.

95. Write an expression with 4 terms that simplifies to $3x - 4$.
96. Write an expression of the form $\frac{____}{____}$ whose product is $6x + 24$.

97. To convert from feet to inches, we multiply by 12. For example, the number of inches in 2 feet is $12 \cdot 2$ inches. If one board has a length of $(x + 2)$ feet and a second board has a length of $(3x - 1)$ inches, express their total length in inches as an algebraic expression.

98. The value of 7 nickels is $5 \cdot 7$ cents. Likewise, the value of $x$ nickels is $5x$ cents. If the money box in a drink machine contains $x$ nickels, $3x$ dimes, and $(30x - 1)$ quarters, express their total value in cents as an algebraic expression.

For Exercises 99 through 104, see the example below.

Example

Simplify $-3xy + 2x^2y - (2xy - 1)$.

Solution

$-3xy + 2x^2y - (2xy - 1)$

$= -3xy + 2x^2y - 2xy + 1 = -5xy + 2x^2y + 1$

Simplify each expression.

99. $5b^2c^3 + 8b^4c^2 - 7b^2c^2$
100. $4m^4p^2 + m^2p^2 - 5m^2p^4$
101. $3x - (2x^2 - 6x) + 7x^2$
102. $9y^2 - (6xy^2 - 5y^2) - 8xy^2$
103. $-(2x^2y + 3z) + 3z - 5x^2y$
104. $-(7c^3d - 8c) - 5c - 4c^3d$

3. Rate your commitment to this course with a number between 1 and 5. Use the diagram below to help.

<table>
<thead>
<tr>
<th>High Commitment</th>
<th>Average Commitment</th>
<th>Not committed at all</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

4. If you have rated your personal commitment level (from the exercise above) as a 1, 2, or 3, list the reasons why this is so. Then determine whether it is possible to increase your commitment level to a 4 or 5.

Good luck, and don’t forget that a positive attitude will make a big difference.
OBJECTIVES
1 Define linear equations and use the addition property of equality to solve linear equations.
2 Use the multiplication property of equality to solve linear equations.
3 Use both properties of equality to solve linear equations.
4 Write word phrases as algebraic expressions.

**OBJECTIVE 1** Defining linear equations and using the addition property. Recall from Section 1.4 that an equation is a statement that two expressions have the same value. Also, a value of the variable that makes an equation a true statement is called a solution or root of the equation. The process of finding the solution of an equation is called solving the equation for the variable. In this section we concentrate on solving linear equations in one variable.

**Linear Equation in One Variable**
A linear equation in one variable can be written in the form

\[ ax + b = c \]

where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

**OBJECTIVES**
1 Define linear equations and use the addition property of equality to solve linear equations.
2 Use the multiplication property of equality to solve linear equations.
3 Use both properties of equality to solve linear equations.
4 Write word phrases as algebraic expressions.

Evaluating a linear equation for a given value of the variable, as we did in Section 1.4, can tell us whether that value is a solution, but we can’t rely on evaluating an equation as our method of solving it.

Instead, to solve a linear equation in \( x \), we write a series of simpler equations, all equivalent to the original equation, so that the final equation has the form

\[ x = \text{number} \quad \text{or} \quad \text{number} = x \]

Equivalent equations are equations that have the same solution. This means that the “number” above is the solution to the original equation.

The first property of equality that helps us write simpler equivalent equations is the addition property of equality.

**Addition Property of Equality**
If \( a, b, \) and \( c \) are real numbers, then

\[ a = b \quad \text{and} \quad a + c = b + c \]

are equivalent equations.

This property guarantees that adding the same number to both sides of an equation does not change the solution of the equation. Since subtraction is defined in terms of addition, we may also subtract the same number from both sides without changing the solution.

A good way to picture a true equation is as a balanced scale. Since it is balanced, each side of the scale weighs the same amount.

If the same weight is added to or subtracted from each side, the scale remains balanced.
We use the addition property of equality to write equivalent equations until the variable is by itself on one side of the equation, and the equation looks like “\( x = \text{number} \)” or “\( \text{number} = x \).”

**EXAMPLE 1** Solve \( x - 7 = 10 \) for \( x \).

**Solution** To solve for \( x \), we want \( x \) alone on one side of the equation. To do this, we add 7 to both sides of the equation.

\[
x - 7 = 10 \\
x - 7 + 7 = 10 + 7 \quad \text{Add 7 to both sides.} \\
x = 17 \quad \text{Simplify.}
\]

The solution of the equation \( x = 17 \) is obviously 17. Since we are writing equivalent equations, the solution of the equation \( x - 7 = 10 \) is also 17.

**Check:** To check, replace \( x \) with 17 in the original equation.

\[
x - 7 = 10 \\
17 - 7 \overset{?}{=} 10 \quad \text{Replace } x \text{ with 17 in the original equation.} \\
10 = 10 \quad \text{True}
\]

Since the statement is true, 17 is the solution.

**PRACTICE 1** Solve: \( x + 3 = -5 \) for \( x \).

**Concept Check** Use the addition property to fill in the blank so that the middle equation simplifies to the last equation.

\[
x - 5 = 3 \\
x - 5 + \_ = 3 + \_ \\
x = 8
\]

**EXAMPLE 2** Solve \( y + 0.6 = -1.0 \) for \( y \).

**Solution** To get \( y \) alone on one side of the equation, subtract 0.6 from both sides of the equation.

\[
y + 0.6 = -1.0 \\
y + 0.6 - 0.6 = -1.0 - 0.6 \quad \text{Subtract 0.6 from both sides.} \\
y = -1.6 \quad \text{Combine like terms.}
\]

**Check:** To check the proposed solution, \(-1.6\), replace \( y \) with \(-1.6\) in the original equation.

\[
y + 0.6 = -1.0 \\
-1.6 + 0.6 \overset{?}{=} -1.0 \quad \text{Replace } y \text{ with } -1.6 \text{ in the original equation.} \\
-1.0 = -1.0 \quad \text{True}
\]

The solution is \(-1.6\).

**PRACTICE 2** Solve: \( y - 0.3 = -2.1 \) for \( y \).

Many times, it is best to simplify one or both sides of an equation before applying the addition property of equality.
CHAPTER 2 Equations, Inequalities, and Problem Solving

Helpful Hint

We may solve an equation so that the variable is alone on either side of the equation. For example, is equivalent to $a = 8$.

If an equation contains parentheses, we use the distributive property to remove them.

EXAMPLE 4 Solve: $7 = -5(2a - 1) - (-11a + 6)$.

Solution

First we simplify both sides of the equation.

Next, we want all terms with a variable on one side of the equation and all numbers on the other side.

Check:

The solution is $-3$.

If an equation contains parentheses, we use the distributive property to remove them.

EXAMPLE 3 Solve: $2x + 3x - 5 + 7 = 10x + 3 - 6x - 4$

Solution

First we simplify both sides of the equation.

Next, we want all terms with a variable on one side of the equation and all numbers on the other side.

Check:

The solution is $-3$.

When solving equations, we may sometimes encounter an equation such as $-x = 5$.

This equation is not solved for $x$ because $x$ is not isolated. One way to solve this equation for $x$ is to recall that

“−” can be read as “the opposite of.”
Section 2.2 The Addition and Multiplication Properties of Equality

We can read the equation \(-x = 5\) then as “the opposite of \(x = 5\).” If the opposite of \(x\) is 5, this means that \(x\) is the opposite of 5 or \(-5\).

In summary,

\[-x = 5 \quad \text{and} \quad x = -5\]

are equivalent equations and \(x = -5\) is solved for \(x\).

**OBJECTIVE 2 Using the multiplication property.** As useful as the addition property of equality is, it cannot help us solve every type of linear equation in one variable. For example, adding or subtracting a value on both sides of the equation does not help solve

\[
\frac{5}{2}x = 15.
\]

Instead, we apply another important property of equality, the **multiplication property of equality**.

**Multiplication Property of Equality**

If \(a\), \(b\), and \(c\) are real numbers and \(c \neq 0\), then

\[a = b \quad \text{and} \quad ac = bc\]

are equivalent equations.

This property guarantees that multiplying both sides of an equation by the same nonzero number does not change the solution of the equation. Since division is defined in terms of multiplication, we may also **divide both sides of the equation by the same nonzero number** without changing the solution.

**EXAMPLE 5** Solve: \(\frac{5}{2}x = 15\).

**Solution** To get \(x\) alone, multiply both sides of the equation by the reciprocal of \(\frac{5}{2}\), which is \(\frac{2}{5}\).

\[
\frac{5}{2}x = 15 \\
\frac{2}{5} \cdot \frac{5}{2}x = \frac{2}{5} \cdot 15 \\
1x = 6 \quad \text{Multiply both sides by} \frac{2}{5}.
\]

or

\[x = 6\]

**Check:** Replace \(x\) with 6 in the original equation.

\[
\frac{5}{2}x = 15 \quad \text{Original equation} \\
\frac{5}{2}(6) = 15 \quad \text{Replace} \ x \ \text{with} \ 6. \\
15 = 15 \quad \text{True}
\]

The solution is 6.
In the equation \( \frac{5}{2}x = 15 \), \( \frac{5}{2} \) is the coefficient of \( x \). When the coefficient of \( x \) is a fraction, we will get \( x \) alone by multiplying by the reciprocal. When the coefficient of \( x \) is an integer or a decimal, it is usually more convenient to divide both sides by the coefficient. (Dividing by a number is, of course, the same as multiplying by the reciprocal of the number.)

**EXAMPLE 6** Solve: \(-3x = 33\)

**Solution** Recall that \(-3x\) means \(-3 \cdot x\). To get \( x \) alone, we divide both sides by the coefficient of \( x \), that is, \(-3\).

\[
\begin{align*}
-3x &= 33 \\
\frac{-3x}{-3} &= \frac{33}{-3} & \text{Divide both sides by } -3. \\
1x &= -11 & \text{Simplify.} \\
x &= -11 \\
\end{align*}
\]

**Check:**

\[
\begin{align*}
-3x &= 33 & \text{Original equation} \\
-3(-11) &= 33 & \text{Replace } x \text{ with } -11. \\
33 &= 33 & \text{True}
\end{align*}
\]

The solution is \(-11\).

**EXAMPLE 7** Solve: \( \frac{y}{7} = 20 \)

**Solution** Recall that \( \frac{y}{7} = \frac{1}{7}y \). To get \( y \) alone, we multiply both sides of the equation by \( \frac{1}{7} \), the reciprocal of \( \frac{1}{7} \).

\[
\begin{align*}
\frac{y}{7} &= 20 \\
\frac{1}{7} \cdot \frac{y}{7} &= 20 & \text{Multiply both sides by } \frac{1}{7}. \\
1y &= 140 & \text{Simplify.} \\
y &= 140 \\
\end{align*}
\]

**Check:**

\[
\begin{align*}
\frac{y}{7} &= 20 & \text{Original equation} \\
\frac{140}{7} &= 20 & \text{Replace } y \text{ with } 140. \\
20 &= 20 & \text{True}
\end{align*}
\]

The solution is 140.

**OBJECTIVE 3** Using both the addition and multiplication properties. Next, we practice solving equations using both properties.
OBJECTIVE 4 | Writing word phrases as algebraic expressions. Next, we practice writing word phrases as algebraic expressions.

EXAMPLE 8 | Solve: $12a - 8a = 10 + 2a - 13 - 7$

**Solution** First, simplify both sides of the equation by combining like terms.

\[
12a - 8a = 10 + 2a - 13 - 7 \\
4a = 2a - 10 \\
\text{Combine like terms.}
\]

To get all terms containing a variable on one side, subtract $2a$ from both sides.

\[
4a - 2a = 2a - 10 - 2a \\
2a = -10 \\
\frac{2a}{2} = \frac{-10}{2} \\
a = -5 \\
\text{Simplify.}
\]

**Check:** Check by replacing $a$ with $-5$ in the original equation. The solution is $-5$. □

PRACTICE 8 | Solve: $6b - 11b = 18 + 2b - 6 + 9$

OBJECTIVE 4 | Writing word phrases as algebraic expressions. Next, we practice writing word phrases as algebraic expressions.

EXAMPLE 9

a. The sum of two numbers is 8. If one number is 3, find the other number.

b. The sum of two numbers is 8. If one number is $x$, write an expression representing the other number.

c. An 8-foot board is cut into two pieces. If one piece is $x$ feet, express the length of the other piece in terms of $x$.

**Solution**

a. If the sum of two numbers is 8 and one number is 3, we find the other number by subtracting 3 from 8. The other number is $8 - 3$ or 5.

b. If the sum of two numbers is 8 and one number is $x$, we find the other number by subtracting $x$ from 8. The other number is represented by $8 - x$.

c. If an 8-foot board is cut into two pieces and one piece is $x$ feet, we find the other length by subtracting $x$ from 8. The other piece is $(8 - x)$ feet.
CHAPTER 2 Equations, Inequalities, and Problem Solving

**PRACTICE**

9. The sum of two numbers is 9. If one number is 2, find the other number.

b. The sum of two numbers is 9. If one number is $x$, write an expression representing the other number.

c. A 9-foot rope is cut into two pieces. If one piece is $x$ feet, express the length of the other piece in terms of $x$.

**EXAMPLE 10** If $x$ is the first of three consecutive integers, express the sum of the three integers in terms of $x$. Simplify if possible.

**Solution** An example of three consecutive integers is

$$7, 8, 9$$

The second consecutive integer is always 1 more than the first, and the third consecutive integer is 2 more than the first. If $x$ is the first of three consecutive integers, the three consecutive integers are

$$x, x+1, x+2$$

Their sum is

$$x + (x+1) + (x+2)$$

In words: first integer + second integer + third integer

Translate: $x + (x+1) + (x+2)$

which simplifies to $3x + 3$.

**PRACTICE** 10. If $x$ is the first of three consecutive even integers, express their sum in terms of $x$.

Below are examples of consecutive even and odd integers.

**Consecutive Even integers:**

$$7, 8, 9, 10, 11, 12, 13$$

**Consecutive Odd integers:**

$$4, 5, 6, 7, 8, 9, 10$$
Helpful Hint

If \( x \) is an odd integer, then \( x + 2 \) is the next odd integer. This 2 simply means that odd integers are always 2 units from each other. (The same is true for even integers. They are always 2 units from each other.)

\[
\begin{array}{cccccc}
-4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array}
\]

\[
\begin{array}{cccccc}
2 \text{ units} & 2 \text{ units} & 2 \text{ units} & 2 \text{ units} \\
\end{array}
\]

## VOCABULARY & READINESS CHECK

Use the choices below to fill in each blank. Some choices will be used more than once.

- addition
- solving
- expression
- true
- multiplication
- equation
- solution
- false

1. The difference between an equation and an expression is that a(n) _____ contains an equal sign, whereas an _____ does not.

2. _____ equations are equations that have the same solution.

3. A value of the variable that makes the equation a true statement is called a(n) _____ of the equation.

4. The process of finding the solution of an equation is called the _____ for the variable.

5. By the _____ property of equality, \( x = -2 \) and \( x + 10 = -2 + 10 \) are equivalent equations.

6. True or false: The equations \( x = \frac{1}{2} \) and \( \frac{x + 1}{2} = x \) are equivalent equations. ______

7. By the _____ property of equality, \( y = \frac{1}{2} \) and \( 5 \cdot \frac{x + 1}{2} = 5 \cdot \frac{1}{2} \) are equivalent equations.

8. True or false: The equations \( \frac{x + 1}{2} = 10 \) and \( 4 \cdot \frac{x + 1}{4} = 10 \) are equivalent equations. ______

9. True or false: The equations \( -7x = 30 \) and \( \frac{-7x}{-7} = \frac{30}{-7} \) are equivalent equations. ______

10. By the _____ property of equality, \( 9x = -63 \) and \( \frac{9x}{9} = \frac{-63}{9} \) are equivalent equations.

Solve each equation mentally.

11. \( 3a = 27 \)  
12. \( 9c = 54 \)  
13. \( 5b = 10 \)  
14. \( 7t = 14 \)
27. \(-x = -12\)  
28. \(-y = 8\)  
29. \(3x + 2x = 50\)  
30. \(-y + 4y = 33\)

Solve each equation. See Examples 5 and 7.

31. \(\frac{2}{3}x = -8\)  
32. \(\frac{3}{4}n = -15\)  
33. \(\frac{1}{6}d = \frac{1}{2}\)  
34. \(\frac{1}{8}n = \frac{1}{4}\)  
35. \(\frac{a}{-2} = 1\)  
36. \(\frac{d}{15} = 2\)  
37. \(\frac{k}{7} = 0\)  
38. \(\frac{f}{-5} = 0\)

39. In your own words, explain the addition property of equality.

40. In your own words, explain the multiplication property of equality.

**MIXED PRACTICE**

Solve each equation. Check each solution. See Examples 1 through 8.

41. \(2x - 4 = 16\)  
42. \(3x - 4 = 26\)  
43. \(-x + 2 = 22\)  
44. \(-x + 4 = -24\)  
45. \(6a + 3 = 3\)  
46. \(8t + 5 = 5\)  
47. \(6t + 10 = -20\)  
48. \(-10y + 15 = 5\)  
49. \(5 - 0.3k = 5\)  
50. \(2 + 0.4p = 2\)  
51. \(-2t + \frac{1}{2}t = -\frac{7}{2}\)  
52. \(-3n - \frac{1}{3} = \frac{8}{3}\)  
53. \(\frac{x}{3} + 2 = -5\)  
54. \(\frac{b}{4} - 1 = -7\)  
55. \(10 = 2x - 1\)  
56. \(12 = 3j - 4\)  
57. \(6x - 8z + 3 = 0\)  
58. \(4a + 1 + a - 11 = 0\)  
59. \(10 - 3x - 6 - 9x = 7\)  
60. \(12x + 30 + 8x - 6 = 10\)  
61. \(\frac{5}{6}t = 10\)  
62. \(\frac{3}{4}x = 9\)  
63. \(1 = 0.4x - 0.6x - 5\)  
64. \(19 = 0.4x - 0.9x - 6\)  
65. \(z - 5z + 7z - 9 = -z\)  
66. \(t - 6t = -13 + t - 3t\)  
67. \(0.4x - 0.6x - 5 = 1\)  
68. \(0.4x - 0.9x - 6 = 19\)  
69. \(6 - 2x + 8 = 10\)  
70. \(-5 - 6y + 6 = 19\)  
71. \(-3a + 6 + 5a = 7a - 8a\)  
72. \(4b - 8 - b = 10b - 3b\)  
73. \(20 = -3(2x + 1) + 7x\)  
74. \(-3 = -5(4x + 3) + 21x\)

See Example 9.

75. Two numbers have a sum of 20. If one number is \(p\), express the other number in terms of \(p\).

76. Two numbers have a sum of 13. If one number is \(y\), express the other number in terms of \(y\).

77. A 10-foot board is cut into two pieces. If one piece is \(x\) feet long, express the other length in terms of \(x\).

78. A 5-foot piece of string is cut into two pieces. If one piece is \(x\) feet long, express the other length in terms of \(x\).

79. Two angles are supplementary if their sum is 180°. If one angle measures \(x°\), express the measure of its supplement in terms of \(x\).

80. Two angles are complementary if their sum is 90°. If one angle measures \(x°\), express the measure of its complement in terms of \(x\).

81. In a mayoral election, April Catarrella received 284 more votes than Charles Pecot. If Charles received \(n\) votes, how many votes did April receive?

82. The length of the top of a computer desk is \(\frac{1}{2}\) feet longer than its width. If its width measures \(m\) feet, express its length as an algebraic expression in \(m\).

83. The Verrazano-Narrows Bridge in New York City is the longest suspension bridge in North America. The Golden Gate Bridge in San Francisco is 60 feet shorter than the Verrazano-Narrows Bridge. If the length of the Verrazano-Narrows Bridge is \(m\) feet, express the length of the Golden Gate Bridge as an algebraic expression in \(m\). (Source: World Almanac, 2000).

84. The longest interstate highway in the U.S. is I-90, which connects Seattle, Washington, and Boston, Massachusetts. The second longest interstate highway, I-80 (connecting San Francisco, California, and Teaneck, New Jersey), is 178.5 miles shorter than I-90. If the length of I-80 is \(m\) miles, express the length of I-90 as an algebraic expression in \(m\).
Section 2.2 The Addition and Multiplication Properties of Equality

85. In a recent election, Pat Ahumada ran against Solomon P. Ortiz for one of Texas’s seats in the U.S. House of Representatives. Ahumada received 47,628 fewer votes than Ortiz. If Ahumada received \( n \) votes, how many did Ortiz receive? (Source: Voter News Service)

86. In a recent U.S. Senate race in Maine, Susan M. Collins received 30,898 more votes than Joseph E. Brennan. If Joseph received \( n \) votes, how many did Susan receive? (Source: Voter News Service)

87. The area of the Sahara Desert in Africa is 7 times the area of the Gobi Desert in Asia. If the area of the Gobi Desert is \( x \) square miles, express the area of the Sahara Desert as an algebraic expression in \( x \).

88. The largest meteorite in the world is the Hoba West located in Namibia. Its weight is 3 times the weight of the Armanty meteorite located in Outer Mongolia. If the weight of the Armanty meteorite is \( y \) kilograms, express the weight of the Hoba West meteorite as an algebraic expression in \( y \).

89. If \( x \) represents the first of two consecutive odd integers, express the sum of the two integers in terms of \( x \).

90. If \( x \) is the first of four consecutive even integers, write their sum as an algebraic expression in \( x \).

91. If \( x \) is the first of four consecutive integers, express the sum of the first integer and the third integer as an algebraic expression containing the variable \( x \).

92. If \( x \) is the first of two consecutive integers, express the sum of 20 and the second consecutive integer as an algebraic expression containing the variable \( x \).

93. Classrooms on one side of the science building are all numbered with consecutive even integers. If the first room on this side of the building is numbered \( x \), write an expression in \( x \) for the sum of five classroom numbers in a row. Then simplify this expression.

94. Two sides of a quadrilateral have the same length, \( x \), while the other two sides have the same length, both being the next consecutive odd integer. Write the sum of these lengths. Then simplify this expression.

**REVIEW AND PREVIEW**

Simplify each expression. See Section 2.1.

95. \( 5x + 2(x - 6) \)

96. \( -7y + 2y - 3(y + 1) \)

97. \( -(x - 1) + x \)

98. \( -(3a - 3) + 2a - 6 \)

Insert \(<, >, \) or \( = \) in the appropriate space to make each statement true. See Sections 1.2 and 1.7.

99. \( (3)^2 \quad -3^2 \)

100. \( (-2)^4 \quad -2^4 \)

101. \( -2^3 \quad -3^1 \)

102. \( (-4)^3 \quad -4^3 \)

**CONCEPT EXTENSIONS**

103. The sum of the angles of a triangle is 180°. If one angle of a triangle measures \( x \)° and a second angle measures \( 2x + 7 \)°, express the measure of the third angle in terms of \( x \). Simplify the expression.

104. A quadrilateral is a four-sided figure like the one shown below whose angle sum is 360°. If one angle measures \( x \)°, a second angle measures \( 3x \)°, and a third angle measures \( 5x \)°, express the measure of the fourth angle in terms of \( x \). Simplify the expression.
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105. Write two terms whose sum is \(-3x\).

106. Write four terms whose sum is \(2y - 6\).

Use the addition property to fill in the blank so that the middle equation simplifies to the last equation. See the Concept Check in this section.

107. \(x - 4 = -9\)
   \[x - 4 + (\quad) = -9 + (\quad)\]
   \(x = -5\)

108. \(a + 9 = 15\)
   \[a + 9 + (\quad) = 15 + (\quad)\]
   \(a = 6\)

Fill in the blanks with numbers of your choice so that each equation has the given solution. Note: Each blank may be replaced with a different number.

109. \(\quad + x = \quad;\) Solution: \(-3\)

110. \(x - \quad = \quad;\) Solution: \(-10\)

111. Let \(x = 1\) and then \(x = 2\) in the equation \(x + 5 = x + 6\). Is either number a solution? How many solutions do you think this equation has? Explain your answer.

112. Let \(x = 1\) and then \(x = 2\) in the equation \(x + 3 = x + 3\). Is either number a solution? How many solutions do you think this equation has? Explain your answer.

Fill in the blank with a number so that each equation has the given solution.

113. \(6x = \quad ;\) solution: \(-8\)

114. \(\quad x = 10;\) solution: \(\frac{4}{7}\)

115. A licensed nurse practitioner is instructed to give a patient 2100 milligrams of an antibiotic over a period of 36 hours. If the antibiotic is to be given every 4 hours starting immediately, how much antibiotic should be given in each dose? To answer this question, solve the equation \(9x = 2100\).

116. Suppose you are a pharmacist and a customer asks you the following question. His child is to receive 13.5 milliliters of a nausea medicine over a period of 54 hours. If the nausea medicine is to be administered every 6 hours starting immediately, how much medicine should be given in each dose?

Use a calculator to determine whether the given value is a solution of the given equation.

117. \(8.13 + 5.85y = 20.05y - 8.91; y = 1.2\)

118. \(3(a + 4.6) = 5a + 2.5; a = 6.3\)

Solve each equation.

119. \(-3.6x = 10.62\)

120. \(4.95y = -31.185\)

121. \(7x - 5.06 = -4.92\)

122. \(0.06y + 2.63 = 2.5562\)

**STUDY SKILLS BUILDER**

How Are Your Homework Assignments Going?

It is very important in mathematics to keep up with homework. Why? Many concepts build on each other. Often your understanding of a day’s concepts depends on an understanding of the previous day’s material.

Remember that completing your homework assignment involves a lot more than attempting a few of the problems assigned.

To complete a homework assignment, remember these four things:

- Attempt all of it.
- Check it.
- Correct it.
- If needed, ask questions about it.

Self-Check

Take a moment and review your completed homework assignments. Answer the questions below based on this review.

1. Approximate the fraction of your homework you have attempted.

2. Approximate the fraction of your homework you have checked (if possible).

3. If you are able to check your homework, have you corrected it when errors have been found?

4. When working homework, if you do not understand a concept, what do you do?
OBJECTIVES

1. Apply a general strategy for solving a linear equation.
2. Solve equations containing fractions.
3. Solve equations containing decimals.
4. Recognize identities and equations with no solution.

2.3 SOLVING LINEAR EQUATIONS

OBJECTIVE 1 Applying a general strategy for solving a linear equation. We now present a general strategy for solving linear equations. One new piece of strategy is a suggestion to “clear an equation of fractions” as a first step. Doing so makes the equation more manageable, since operating on integers is more convenient than operating on fractions.

Solving Linear Equations in One Variable

STEP 1. Multiply on both sides by the LCD to clear the equation of fractions if they occur.
STEP 2. Use the distributive property to remove parentheses if they occur.
STEP 3. Simplify each side of the equation by combining like terms.
STEP 4. Get all variable terms on one side and all numbers on the other side by using the addition property of equality.
STEP 5. Get the variable alone by using the multiplication property of equality.
STEP 6. Check the solution by substituting it into the original equation.

EXAMPLE 1 Solve: $4(2x - 3) + 7 = 3x + 5$

Solution There are no fractions, so we begin with Step 2.

STEP 2. $4(2x - 3) + 7 = 3x + 5$ Apply the distributive property.

STEP 3. $8x - 12 + 7 = 3x + 5$ Combine like terms.

STEP 4. Get all variable terms on the same side of the equation by subtracting $3x$ from both sides, then adding 5 to both sides.

\[
\begin{align*}
8x - 5 &= 3x + 5 - 3x \\
5x &= 10
\end{align*}
\]

STEP 5. Use the multiplication property of equality to get $x$ alone.

\[
\frac{5x}{5} = \frac{10}{5} \quad \text{Divide both sides by 5.}
\]

\[
x = 2 \quad \text{Simplify.}
\]

STEP 6. Check.

\[
\begin{align*}
4(2x - 3) + 7 &= 3x + 5 \\ 4[2(2) - 3] + 7 &= 3(2) + 5 \\
4(4 - 3) + 7 &= 6 + 5 \\
4(1) + 7 &= 11 \\
4 + 7 &= 11 \\
11 &= 11 \quad \text{True}
\end{align*}
\]

The solution is 2 or the solution set is \{2\}.

PRACTICE 1 Solve: $2(4a - 9) + 3 = 5a - 6$
EXAMPLE 2  Solve: \(8(2 - t) = -5t\)

**Solution**  First, we apply the distributive property.

\[
8(2 - t) = -5t
\]

**STEP 2.**  \(16 - 8t = -5t\)  Use the distributive property.

**STEP 4.**  \(16 - 8t + 8t = -5t + 8t\)  To get variable terms on one side, add 8t to both sides.

\[
16 = 3t
\]

**STEP 5.**  \(\frac{16}{3} = \frac{3t}{3}\)  Divide both sides by 3.

\[
\frac{16}{3} = t
\]

**STEP 6.**  Check.

\[
\begin{align*}
8(2 - t) &= -5t & \text{Original equation} \\
8\left(2 - \frac{16}{3}\right) &= -5\left(\frac{16}{3}\right) & \text{Replace } t \text{ with } \frac{16}{3} \\
8\left(\frac{6}{3} - \frac{16}{3}\right) &= -\frac{80}{3} & \text{The LCD is 3.} \\
-\frac{80}{3} &= -\frac{80}{3} & \text{Subtract fractions.}
\end{align*}
\]

The solution is \(\frac{16}{3}\).  \(\square\)

PRACTICE 2 Solve: \(7(x - 3) = -6x\)

OBJECTIVE 2  Solving equations containing fractions. If an equation contains fractions, we can clear the equation of fractions by multiplying both sides by the LCD of all denominators. By doing this, we avoid working with time-consuming fractions.

EXAMPLE 3  Solve: \(\frac{x}{2} - 1 = \frac{2}{3}x - 3\)

**Solution**  We begin by clearing fractions. To do this, we multiply both sides of the equation by the LCD of 2 and 3, which is 6.

\[
\frac{x}{2} - 1 = \frac{2}{3}x - 3
\]

**STEP 1.**  \(6\left(\frac{x}{2} - 1\right) = 6\left(\frac{2}{3}x - 3\right)\)  Multiply both sides by the LCD, 6.

**STEP 2.**  \(\left(\frac{x}{2}\right) - 6(1) = \left(\frac{2}{3}x\right) - 6(3)\)  Apply the distributive property.

\[
3x - 6 = 4x - 18
\]

Simplify.
There are no longer grouping symbols and no like terms on either side of the equation, so we continue with Step 4.

\[3x - 6 = 4x - 18\]  

**STEP 4.**  
\[3x - 6 - 3x = 4x - 18 - 3x\]  
\[-6 = x - 18\]  
\[-6 + 18 = x - 18 + 18\]  
\[12 = x\]  

**STEP 5.** The variable is now alone, so there is no need to apply the multiplication property of equality.

**STEP 6.** Check.

\[\frac{x}{2} - 1 = \frac{2}{3}x - 3\]  
\[12\]  
\[6 - 1 \neq 8 - 3\]  
\[5 = 5\]  

The solution is 12.

**EXAMPLE 4**  
Solve:  
\[\frac{2(a + 3)}{3} = 6a + 2\]

**Solution.** We clear the equation of fractions first.

\[\frac{2(a + 3)}{3} = 6a + 2\]  
\[3 \cdot \frac{2(a + 3)}{3} = 3(6a + 2)\]  
\[2(a + 3) = 3(6a + 2)\]  

**STEP 1.** Clear the fraction by multiplying both sides by the LCD, 3.

**STEP 2.** Next, we use the distributive property and remove parentheses.

\[2a + 6 = 18a + 6\]  
\[2a = 18a\]  
\[2a - 18a = 18a - 18a\]  
\[-16a = 0\]  
\[\frac{-16a}{-16} = \frac{0}{-16}\]  
\[a = 0\]  

**STEP 5.** Divide both sides by -16.

**STEP 6.** To check, replace \(a\) with 0 in the original equation. The solution is 0.

**OBJECTIVE 3**  
Solving equations containing decimals. When solving a problem about money, you may need to solve an equation containing decimals. If you choose, you may multiply to clear the equation of decimals.
OBJECTIVE 4 Recognizing identities and equations with no solution. So far, each equation that we have solved has had a single solution. However, not every equation in one variable has a single solution. Some equations have no solution, while others have an infinite number of solutions. For example, 

\[ x + 5 = x + 7 \]

has no solution since no matter which real number we replace \( x \) with, the equation is false.

\[
\text{real number} + 5 = \text{same real number} + 7 \quad \text{FALSE}
\]

On the other hand,

\[ x + 6 = x + 6 \]

has infinitely many solutions since \( x \) can be replaced by any real number and the equation is always true.

\[
\text{real number} + 6 = \text{same real number} + 6 \quad \text{TRUE}
\]

The equation \( x + 6 = x + 6 \) is called an identity. The next few examples illustrate special equations like these.

EXAMPLE 5 Solve: \( 0.25x + 0.10(x - 3) = 0.05(22) \)

**Solution** First we clear this equation of decimals by multiplying both sides of the equation by 100. Recall that multiplying a decimal number by 100 has the effect of moving the decimal point 2 places to the right.

\[
0.25x + 0.10(x - 3) = 0.05(22)
\]

**STEP 1.**

\[
25x + 10(x - 3) = 5(22)
\]

Apply the distributive property.

**STEP 2.**

\[
25x + 10x - 30 = 110
\]

Combine like terms.

**STEP 3.**

\[
35x - 30 = 110
\]

Combine like terms.

**STEP 4.**

\[
35x - 30 + 30 = 110 + 30
\]

Add 30 to both sides.

\[
35x = 140
\]

Combine like terms.

**STEP 5.**

\[
\frac{35x}{35} = \frac{140}{35}
\]

Divide both sides by 35.

\[
x = 4
\]

**STEP 6.** To check, replace \( x \) with 4 in the original equation. The solution is 4.

EXAMPLE 6 Solve: \( 4(x + 4) - x = 2(x + 11) + x \)

**Solution** Apply the distributive property on both sides.

\[
-2(x - 5) + 10 = -3(x + 2) + x
\]

Apply the distributive property on both sides.

\[
-2x + 10 + 10 = -3x - 6 + x
\]

Apply the distributive property on both sides.

\[
-2x + 20 = -2x - 6
\]

Combine like terms.

\[
-2x + 20 + 2x = -2x - 6 + 2x
\]

Add 2x to both sides.

\[
20 = -6
\]

Combine like terms.

The final equation contains no variable terms, and there is no value for \( x \) that makes \( 20 = -6 \) a true equation. We conclude that there is no solution to this equation. In set notation, we can indicate that there is no solution with the empty set, \( \{ \} \), or use the empty set or null set symbol, \( \emptyset \). In this chapter, we will simply write no solution.

PRACTICE 6 Solve: \( 4(x + 4) - x = 2(x + 11) + x \)
Example 7

Solve: \(3(x - 4) = 3x - 12\)

Solution

\[
3(x - 4) = 3x - 12
\]

Apply the distributive property.

The left side of the equation is now identical to the right side. Every real number may be substituted for \(x\) and a true statement will result. We arrive at the same conclusion if we continue.

\[
egin{align*}
3x - 12 &= 3x - 12 & \text{Apply the distributive property.} \\
3x - 12 + 12 &= 3x - 12 + 12 & \text{Add 12 to both sides.} \\
3x &= 3x & \text{Combine like terms.} \\
3x - 3x &= 3x - 3x & \text{Subtract 3x from both sides.} \\
0 &= 0
\end{align*}
\]

Again, one side of the equation is identical to the other side. Thus, \(3(x - 4) = 3x - 12\) is an identity and all real numbers are solutions. In set notation, this is \(\{\text{all real numbers}\}\).

Practice 7

Solve: \(12x - 18 = 9(x - 2) + 3x\)

Answers to Concept Check:

a. Every real number is a solution.

b. The solution is 0.

c. There is no solution.

Concept Check

Suppose you have simplified several equations and obtain the following results. What can you conclude about the solutions to the original equation?

a. \(x = 7\)

b. \(x = 0\)

c. \(x = -4\)

Calculator Explorations

Checking Equations

We can use a calculator to check possible solutions of equations. To do this, replace the variable by the possible solution and evaluate both sides of the equation separately.

\[
\begin{align*}
\text{Equation:} & \quad 3x - 4 = 2(x + 6) & \text{Solution:} & \quad x = 16 \\
3x - 4 &= 2(x + 6) & \text{Original equation} \\
3(16) - 4 &= 2(16 + 6) & \text{Replace \(x\) with 16.}
\end{align*}
\]

Now evaluate each side with your calculator.

Evaluate left side:

\[
\begin{align*}
3 \times 16 - 4 & = \text{ or } \text{ENTER} \quad \text{Display: } 44 \text{ or } 3 \times 16 - 4 \\
\end{align*}
\]

Evaluate right side:

\[
\begin{align*}
2 (16 + 6) & = \text{ or } \text{ENTER} \quad \text{Display: } 44 \text{ or } 2(16 + 6)
\end{align*}
\]

Since the left side equals the right side, the equation checks.

Use a calculator to check the possible solutions to each equation.

1. \(2x = 48 + 6x; \quad x = -12\)

2. \(-3x - 7 = 3x - 1; \quad x = -1\)

3. \(5x - 2.6 = 2(x + 0.8); \quad x = 4.4\)

4. \(-1.6x - 3.9 = -6.9x - 25.6; \quad x = 5\)

5. \(\frac{564x}{4} = 200x - 11(649); \quad x = 121\)

6. \(20(x - 39) = 5x - 432; \quad x = 23.2\)
VOCABULARY & READINESS CHECK

Throughout algebra, it is important to be able to identify equations and expressions.

Remember,
• an equation contains an equals sign and
• an expression does not.

Among other things,
• we solve equations and
• we simplify or perform operations on expressions.

Identify each as an equation or an expression.

1. \( x = -7 \)  
2. \( 4y - 6 = 9y + 1 \)  
3. \( \frac{1}{x} - \frac{x - 1}{8} = 6 \)  
4. \( \frac{1}{x} - \frac{x - 1}{8} = 6 \)  
5. \( 0.1x + 9 = 0.2x \)  
6. \( 0.1x^2 + 9y - 0.2x^2 \)

Solve each equation. See Examples 1 and 2.

1. \(-4y + 10 = -2(3y + 1)\)  
2. \(-3x + 1 = -2(4x + 2)\)  
3. \(15x - 8 = 10 + 9x\)  
4. \(15x - 5 = 7 + 12x\)  
5. \(-2(3x - 4) = 2x\)  
6. \(-5(x - 10) = 5x\)  

Solve each equation. See Examples 3 through 5.

7. \(5x - 1 = 2(3x) = 1\)  
8. \(3(2 - 5x) + 4(6x) = 12\)  
9. \(-6x - 3 - 26 = -8\)  
10. \(-4(x - 4) - 23 = -7\)  
11. \(8 - 2(a + 1) = 9 + a\)  
12. \(5 - 6(2 + b) = b - 14\)  
13. \(4x + 3 = -3 + 2x + 14\)  
14. \(6y - 8 = -6 + 3y + 13\)  
15. \(-2y - 10 = 5y + 18\)  
16. \(-7n + 5 = 8n - 10\)

Solve each equation. See Examples 6 and 7.

29. \(4(3x + 2) = 12x + 8\)  
30. \(14x + 7 = 7(2x + 1)\)  
31. \(\frac{x}{4} + 1 = \frac{x}{4}\)  
32. \(\frac{x}{3} - 2 = \frac{x}{3}\)  
33. \(3x - 7 = 3(x + 1)\)  
34. \(2(x + 5) = 2x + 10\)  
35. \(-2(6x - 5) + 4 = -12x + 14\)  
36. \(-5(4y - 3) + 2 = -20y + 17\)

MIXED PRACTICE

Solve. See Examples 1 through 7.

37. \(\frac{6(3 - z)}{5} = -z\)  
38. \(\frac{4(5 - w)}{3} = -w\)  
39. \(-3(2r - 5) + 2r = 5t - 4\)  
40. \(-(4a - 7) - 5a = 10 + a\)  
41. \(5y + 2(y - 6) = 4(y + 1) - 2\)  
42. \(9x + 3(x - 4) = 10(x - 5) + 7\)  
43. \(\frac{3(x - 5)}{2} = \frac{2(x + 5)}{3}\)  
44. \(\frac{5(x - 1)}{4} = \frac{3(x + 1)}{2}\)  
45. \(0.7x - 2.3 = 0.5\)  
46. \(0.9x - 4.1 = 0.4\)  
47. \(5x - 5 = 2(x + 1) + 3x - 7\)

Solve each equation. See Examples 6 and 7.

29. \(4(3x + 2) = 12x + 8\)  
30. \(14x + 7 = 7(2x + 1)\)  
31. \(\frac{x}{4} + 1 = \frac{x}{4}\)  
32. \(\frac{x}{3} - 2 = \frac{x}{3}\)  
33. \(3x - 7 = 3(x + 1)\)  
34. \(2(x + 5) = 2x + 10\)  
35. \(-2(6x - 5) + 4 = -12x + 14\)  
36. \(-5(4y - 3) + 2 = -20y + 17\)

MIXED PRACTICE

Solve. See Examples 1 through 7.

37. \(\frac{6(3 - z)}{5} = -z\)  
38. \(\frac{4(5 - w)}{3} = -w\)  
39. \(-3(2r - 5) + 2r = 5t - 4\)  
40. \(-(4a - 7) - 5a = 10 + a\)  
41. \(5y + 2(y - 6) = 4(y + 1) - 2\)  
42. \(9x + 3(x - 4) = 10(x - 5) + 7\)  
43. \(\frac{3(x - 5)}{2} = \frac{2(x + 5)}{3}\)  
44. \(\frac{5(x - 1)}{4} = \frac{3(x + 1)}{2}\)  
45. \(0.7x - 2.3 = 0.5\)  
46. \(0.9x - 4.1 = 0.4\)  
47. \(5x - 5 = 2(x + 1) + 3x - 7\)
48. \(3(2x - 1) + 5 = 6x + 2\)
49. \(4(2n + 1) = 3(6n + 3) + 1\)
50. \(4(4y + 2) = 2(1 + 6y) + 8\)
51. \(x + \frac{5}{4} = \frac{3}{4}x\)
52. \(\frac{7}{8}x + \frac{1}{4} = \frac{3}{4}x\)
53. \(\frac{x}{2} - 1 = \frac{x}{5} + 2\)
54. \(\frac{x}{5} - 7 = \frac{x}{3} - 5\)
55. \(2(x + 3) - 5 = 5x - 3(1 + x)\)
56. \(4(2 + x) + 1 = 7x - 3(x - 2)\)
57. \(0.06 - 0.01(x + 1) = -0.02(2 - x)\)
58. \(-0.01(5x + 4) = 0.04 = 0.01(x + 4)\)
59. \(\frac{9}{2} + \frac{5}{2}y = 2y - 4\)
60. \(3 - \frac{1}{2}x = 5x - 8\)
61. \(-2y - 10 = 5y + 18\)
62. \(7n + 5 = 10n - 10\)
63. \(0.6x - 0.1 = 0.5x + 0.2\)
64. \(0.2x - 0.1 = 0.6x - 2.1\)
65. \(0.02(6t - 3) = 0.12(t - 2) + 0.18\)
66. \(0.03(2m + 7) = 0.06(5 + m) - 0.09\)

**CONCEPT EXTENSIONS**

**See the Concept Check in this section.**

75. a. Solve: \(x + 3 = x + 3\)
   b. If you simplify an equation and get 0 = 0, what can you conclude about the solution(s) of the original equation?
   c. On your own, construct an equation for which every real number is a solution.

76. a. Solve: \(x + 3 = x + 5\)
   b. If you simplify an equation and get 3 = 5, what can you conclude about the solution(s) of the original equation?
   c. On your own, construct an equation that has no solution.

**REVIEW AND PREVIEW**

Write each phrase as an algebraic expression. Use \(x\) for the unknown number. See Section 2.1.

67. A number subtracted from \(-8\)
68. Three times a number
69. The sum of \(-3\) and twice a number
70. The difference of 8 and twice a number
71. The product of 9 and the sum of a number and 20
72. The quotient of \(-12\) and the difference of a number and 3

See Section 2.1.

73. A plot of land is in the shape of a triangle. If one side is \(x\) meters, a second side is \((2x - 3)\) meters and a third side is \((3x - 5)\) meters, express the perimeter of the lot as a simplified expression in \(x\).

74. A portion of a board has length \(x\) feet. The other part has length \((7x - 9)\) feet. Express the total length of the board as a simplified expression in \(x\).

\[x\] feet \[S\]

\[(7x - 9)\] feet

\(\text{?}\)

\(\text{x centimeters}\)

\(\text{2x centimeters}\)

\(\text{x centimeters}\)

\(\text{2x centimeters}\)
100  CHAPTER 2  Equations, Inequalities, and Problem Solving

\[ 86. \] The perimeter of the following triangle is 35 meters.

\[
\begin{align*}
\text{\(x\) meters} & \quad \text{\((2x + 1)\) meters} \\
\text{\((3x - 2)\) meters} & \quad \text{\(87. \ x + ____ = 2x - ____; \ \text{solution: 9}\)} \\
\text{\(88. \ -5x - ____ = ____; \ \text{solution: 2}\)} \\
\end{align*}
\]

**Solve.**

\[
\begin{align*}
\text{\(89. \ 1000(7x - 10) = 50(412 + 100x)\)} \\
\text{\(90. \ 1000(x + 40) = 100(16 + 7x)\)} \\
\text{\(91. \ 0.035x + 5.112 = 0.010x + 5.107\)} \\
\text{\(92. \ 0.127x - 2.685 = 0.027x - 2.38\)} \\
\end{align*}
\]

---

**THE BIGGER PICTURE  SIMPLIFYING EXPRESSIONS AND SOLVING EQUATIONS**

Now we continue our outline started in Section 1.7. Although suggestions are given, this outline should be in your own words. Once you complete this new portion, try the exercises to the right.

I. Simplifying Expressions
   A. Real Numbers
      1. Add (Section 1.5)
      2. Subtract (Section 1.6)
      3. Multiply or Divide (Section 1.7)

II. Solving Equations
   A. Linear Equations: power on variable is 1 and there are no variables in the denominator
      \[
      \begin{align*}
      7(x - 3) &= 4x + 6 \quad \text{Linear equation. Simplify both sides, then get variable terms on one side, numbers on the other side.} \\
      7x - 21 &= 4x + 6 \quad \text{Use the distributive property.} \\
      7x &= 4x + 27 \quad \text{Add 21 to both sides.} \\
      3x &= 27 \quad \text{Subtract 4x from both sides.} \\
      x &= 9 \quad \text{Divide both sides by 3.}
      \end{align*}
      \]

For Exercises 93 through 96, see the example below.

**Example**

Solve: \(t(t + 4) = t^2 - 2t + 6\).

**Solution**

\[
\begin{align*}
\text{\(t(t + 4) = t^2 - 2t + 6\)} \\
\text{\(t^2 + 4t = t^2 - 2t + 6\)} \\
\text{\(t^2 + 4t - t^2 = t^2 - 2t + 6 - t^2\)} \\
\text{\(4t = -2t + 6\)} \\
\text{\(4t + 2t = -2t + 6 + 2t\)} \\
\text{\(6t = 6\)} \\
\text{\(t = 1\)}
\end{align*}
\]

**Solve each equation.**

\[
\begin{align*}
\text{\(93. \ x(x - 3) = x^2 + 5x + 7\)} \\
\text{\(94. \ t^2 - 6t = t(8 + t)\)} \\
\text{\(95. \ 2z(z + 6) = 2z^2 + 12z - 8\)} \\
\text{\(96. \ y^2 - 4y + 10 = y(y - 5)\)}
\end{align*}
\]
INTEGRATED REVIEW  SOLVING LINEAR EQUATIONS

Sections 2.1–2.3

Solve. Feel free to use the steps given in Section 2.3.

1. \( x - 10 = -4 \)
2. \( y + 14 = -3 \)
3. \( 9y = 108 \)
4. \( -3x = 78 \)
5. \( -6x + 7 = 25 \)
6. \( 5y - 42 = -47 \)
7. \( \frac{2}{3}x = 9 \)
8. \( \frac{4}{5}z = 10 \)
9. \( \frac{r}{-4} = -2 \)
10. \( y = 8 \)
11. \( 6 - 2x + 8 = 10 \)
12. \( -5 - 6y + 6 = 19 \)
13. \( 2x - 7 = 2x - 27 \)
14. \( 3 + 8y = 8y - 2 \)
15. \( -3a + 6 + 5a = 7a - 8a \)
16. \( 4b - 8 - b = 10b - 3b \)
17. \( -\frac{2}{3}x = \frac{5}{9} \)
18. \( \frac{3}{8}y = -\frac{1}{16} \)
19. \( 10 = -6n + 16 \)
20. \( -5 = -2m + 7 \)
21. \( 3(5c - 1) - 2 = 13c + 3 \)
22. \( 4(3t + 4) - 20 = 3 + 5t \)
23. \( \frac{2(z + 3)}{3} = 5 - z \)
24. \( \frac{3(w + 2)}{4} = 2w + 3 \)
25. \( -2(2x - 5) = -3x + 7 - x + 3 \)
26. \( -4(5x - 2) = -12x + 4 - 8x + 4 \)
27. \( 0.02(6t - 3) = 0.04(t - 2) + 0.02 \)
28. \( 0.03(m + 7) = 0.02(5 - m) + 0.03 \)
29. \( -3y = \frac{4(y - 1)}{5} \)
30. \( -4x = \frac{5(1 - x)}{6} \)
31. \( \frac{5}{3}x - \frac{7}{3} = x \)
32. \( \frac{7}{5}n + \frac{3}{5} = -n \)
33. \( \frac{1}{10}(3x - 7) = \frac{3}{10}x + 5 \)
34. \( \frac{1}{7}(2x - 5) = \frac{2}{7}x + 1 \)
35. \( 5 + 2(3x - 6) = -4(6x - 7) \)
36. \( 3 + 5(2x - 4) = -7(5x + 2) \)

2.4 AN INTRODUCTION TO PROBLEM SOLVING

OBJECTIVES

Apply the steps for problem solving as we

1. Solve problems involving direct translations.
2. Solve problems involving relationships among unknown quantities.
3. Solve problems involving consecutive integers.

OBJECTIVE 1  Solving direct translation problems. In previous sections, you practiced writing word phrases and sentences as algebraic expressions and equations to help prepare for problem solving. We now use these translations to help write equations that model a problem. The problem-solving steps given next may be helpful.

General Strategy for Problem Solving

1. UNDERSTAND the problem. During this step, become comfortable with the problem. Some ways of doing this are:
   - Read and reread the problem.
   - Choose a variable to represent the unknown.
   - Construct a drawing, whenever possible.
   - Propose a solution and check. Pay careful attention to how you check your proposed solution. This will help when writing an equation to model the problem.

2. TRANSLATE the problem into an equation.
3. SOLVE the equation.
4. INTERPRET the results: Check the proposed solution in the stated problem and state your conclusion.

Much of problem solving involves a direct translation from a sentence to an equation.
EXAMPLE 1  Finding an Unknown Number

Twice a number, added to seven, is the same as three subtracted from the number. Find the number.

Solution  Translate the sentence into an equation and solve.

In words: \( \text{twice a number} \) \( + \) \( \text{seven} \) \( = \) \( \text{three subtracted from the number} \)

Translate: \( 2x + 7 = x - 3 \)

To solve, begin by subtracting \( x \) from both sides to isolate the variable term.

\[
2x + 7 = x - 3 \\
2x + 7 - x = x - 3 - x \\
x + 7 = -3 \\
x + 7 - 7 = -3 - 7 \\
x = -10
\]

Combine like terms.
Subtract \( x \) from both sides.
Subtract 7 from both sides.
Combine like terms.

Check the solution in the problem as it was originally stated. To do so, replace “number” in the sentence with “Twice added to 7 is the same as 3 subtracted from the number.”

\[
2(-10) + 7 = -10 - 3 \\
-13 = -13
\]

The unknown number is \(-10\).

PRACTICE 1  Three times a number, minus 6, is the same as two times a number, plus 3. Find the number.

Helpful Hint
When checking solutions, go back to the original stated problem, rather than to your equation in case errors have been made in translating to an equation.

EXAMPLE 2  Finding an Unknown Number

Twice the sum of a number and 4 is the same as four times the number, decreased by 12. Find the number.

Solution

1. UNDERSTAND. Read and reread the problem. If we let \( x \) = the unknown number, then

   “the sum of a number and 4” translates to “\( x + 4 \)” and “four times the number” translates to “\( 4x \).”

2. TRANSLATE.

\[
\text{twice} \quad \text{sum of a number and 4} \quad \text{is the same as} \quad \text{four times the number} \quad \text{decreased by} \quad 12
\]

\[
\downarrow \quad (x + 4) \quad = \quad 4x \quad - \quad 12
\]
3. Solve.

\[2(x + 4) = 4x - 12\]
\[2x + 8 = 4x - 12\] Apply the distributive property.
\[2x + 8 - 4x = 4x - 12 - 4x\] Subtract 4x from both sides.
\[-2x + 8 = -12\]
\[-2x + 8 - 8 = -12 - 8\] Subtract 8 from both sides.
\[-2x = -20\]
\[-2x \div -2 = -20 \div -2\] Divide both sides by -2.
\[x = 10\]

4. Interpret.

Check: Check this solution in the problem as it was originally stated. To do so, replace “number” with 10. Twice the sum of “10” and 4 is 28, which is the same as 4 times “10” decreased by 12.

State: The number is 10.

**Example 3** Finding the Length of a Board

Balsa wood sticks are commonly used for building models (for example, bridge models). A 48-inch Balsa wood stick is to be cut into two pieces so that the longer piece is 3 times the shorter. Find the length of each piece.

**Solution**

1. Understand the problem. To do so, read and reread the problem. You may also want to propose a solution. For example, if 10 inches represents the length of the shorter piece, then 3(10) = 30 inches is the length of the longer piece, since it is 3 times the length of the shorter piece. This guess gives a total board length of 10 inches + 30 inches = 40 inches, too short. However, the purpose of proposing a solution is not to guess correctly, but to help better understand the problem and how to model it.

Since the length of the longer piece is given in terms of the length of the shorter piece, let’s let

\[x = \text{length of shorter piece, then}\]
\[3x = \text{length of longer piece}\]
2. TRANSLATE the problem. First, we write the equation in words.

<table>
<thead>
<tr>
<th>length of shorter piece</th>
<th>added to</th>
<th>length of longer piece</th>
<th>equals</th>
<th>total length of board</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$+$</td>
<td>$3x$</td>
<td>$=$</td>
<td>48</td>
</tr>
</tbody>
</table>

3. SOLVE.

\[ x + 3x = 48 \]
\[ 4x = 48 \]
\[ x = \frac{48}{4} \]
\[ x = 12 \]

4. INTERPRET.

Check: Check the solution in the stated problem. If the shorter piece of board is 12 inches, the longer piece is $3 \times 12 = 36$ inches and the sum of the two pieces is 12 inches + 36 inches = 48 inches.

State: The shorter piece of Balsa wood is 12 inches and the longer piece of Balsa wood is 36 inches.

Example 4  Finding the Number of Democratic and Republican Representatives

In a recent year, the U.S. House of Representatives had a total of 435 Democrats and Republicans. There were 31 more Democratic representatives than Republican representatives. Find the number of representatives from each party. (Source: Office of the Clerk of the U.S. House of Representatives)

Solution

1. UNDERSTAND. Read and reread the problem. Let’s suppose that there were 200 Republican representatives. Since there were 31 more Democrats than Republicans, there must have been $200 + 31 = 231$ Democrats. The total number of Democrats and Republicans was then $200 + 231 = 431$. This is incorrect since the total should be 435, but now we have a better understanding of the problem.

In general, if we let $x = \text{number of Republicans}$, then

\[ x + 31 = \text{number of Democrats} \]

2. TRANSLATE. First we write the equation in words.

<table>
<thead>
<tr>
<th>Number of Republicans</th>
<th>added to</th>
<th>number of Democrats</th>
<th>equals</th>
<th>435</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$+$</td>
<td>$(x + 31)$</td>
<td>$=$</td>
<td>435</td>
</tr>
</tbody>
</table>

3. SOLVE.

\[ x + (x + 31) = 435 \]
\[ 2x + 31 = 435 \]
\[ 2x = 404 \]
\[ x = \frac{404}{2} \]
\[ x = 202 \]
4. INTERPRET.

Check: If there were 202 Republican representatives, then there were 202 + 31 = 233 Democratic representatives. The total number of Democratic or Republican representatives is 202 + 233 = 435. The results check.

State: There were 202 Republican and 233 Democratic representatives in Congress.

Practice

In a recent year, there were 6 more Democratic State Governors than Republican State Governors. Find the number of State Governors from each party. (We are only counting the 50 states.) (Source: National Conference of State Legislatures).

Example 5 Finding Angle Measures

If the two walls of the Vietnam Veterans Memorial in Washington, D.C., were connected, an isosceles triangle would be formed. The measure of the third angle is 97.5° more than the measure of either of the other two equal angles. Find the measure of the third angle. (Source: National Park Service)

Solution

1. UNDERSTAND. Read and reread the problem. We then draw a diagram (recall that an isosceles triangle has two angles with the same measure) and let

   \[ x = \text{degree measure of one angle} \]
   \[ x = \text{degree measure of the second equal angle} \]
   \[ x + 97.5 = \text{degree measure of the third angle} \]

2. TRANSLATE. Recall that the sum of the measures of the angles of a triangle equals 180.

   \[
   \begin{array}{c|c|c|c}
   \text{measure of first angle} & \text{measure of second angle} & \text{measure of third angle} & \text{equals} \\
   \hline
   x & x & (x + 97.5) & 180 \\
   \hline
   \end{array}
   \]

3. SOLVE.

   \[
   \begin{align*}
   x + x + (x + 97.5) &= 180 \\
   3x + 97.5 &= 180 & \text{Combine like terms.} \\
   3x + 97.5 - 97.5 &= 180 - 97.5 & \text{Subtract 97.5 from both sides.} \\
   3x &= 82.5 \\
   \frac{3x}{3} &= \frac{82.5}{3} & \text{Divide both sides by 3.} \\
   x &= 27.5
   \end{align*}
   \]

4. INTERPRET.

Check: If \( x = 27.5 \), then the measure of the third angle is \( x + 97.5 = 125 \). The sum of the angles is then \( 27.5 + 27.5 + 125 = 180 \), the correct sum.

State: The third angle measures 125°.*

Practice

The second angle of a triangle measures three times as large as the first. If the third angle measures 55° more than the first, find the measures of all three angles.

*The two walls actually meet at an angle of 125 degrees 12 minutes. The measurement of 97.5° given in the problem is an approximation.
OBJECTIVE 3 Solving consecutive integer problems. The next example has to do with consecutive integers. Recall what we have learned thus far about these integers.

<table>
<thead>
<tr>
<th>Example</th>
<th>General Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consecutive Integers</td>
<td>11, 12, 13 Let ( x ) be an integer. ( x, x + 1, x + 2 )</td>
</tr>
<tr>
<td>Consecutive Even Integers</td>
<td>38, 40, 42 Let ( x ) be an even integer. ( x, x + 2, x + 4 )</td>
</tr>
<tr>
<td>Consecutive Odd Integers</td>
<td>57, 59, 61 Let ( x ) be an odd integer. ( x, x + 2, x + 4 )</td>
</tr>
</tbody>
</table>

**EXAMPLE 6** Some states have a single area code for the entire state. Two such states have area codes that are consecutive odd integers. If the sum of these integers is 1208, find the two area codes. *(Source: North American Numbering Plan Administration)*

**Solution:**

1. **UNDERSTAND.** Read and reread the problem. If we let
   
   \[ x = \text{the first odd integer, then} \]
   
   \[ x + 2 = \text{the next odd integer} \]

2. **TRANSLATE.**

   \[ \begin{array}{c|c|c|c|c|c}
   \text{first odd integer} & \text{the sum of} & \text{next odd integer} & \text{is} & 1208 \\
   \hline
   x & + & (x + 2) & = & 1208 \\
   \end{array} \]

3. **SOLVE.**

   \[
   \begin{align*}
   x + x + 2 &= 1208 \\
   2x + 2 &= 1208 \\
   2x &= 1206 \\
   x &= 603 
   \end{align*}
   \]

4. **INTERPRET.**

   **Check:** If \( x = 603 \), then the next odd integer \( x + 2 = 603 + 2 = 605 \). Notice their sum, \( 603 + 605 = 1208 \), as needed.

   **State:** The area codes are 603 and 605.

   **Note:** New Hampshire’s area code is 603 and South Dakota’s area code is 605.

**PRACTICE**

6. The sum of three consecutive even integers is 144. Find the integers.
VOCABULARY & READINESS CHECK

Fill in the table.

<table>
<thead>
<tr>
<th></th>
<th>A number:</th>
<th></th>
<th>Double the number:</th>
<th></th>
<th>Double the number, decreased by 31:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write each of the following as equations. Then solve. See Examples 1 and 2.

1. The sum of twice a number, and 7, is equal to the sum of a number and 6. Find the number.
2. The difference of three times a number, and 1, is the same as twice a number. Find the number.
3. Three times a number, minus 6, is equal to two times a number, plus 8. Find the number.
4. The sum of 4 times a number, and −2, is equal to the sum of 5 times a number, and −2. Find the number.
5. Twice the difference of a number and 8 is equal to three times the sum of the number and 3. Find the number.
6. Five times the sum of a number and −1 is the same as 6 times the number. Find the number.
7. Four times the sum of −2 and a number is the same as five times the number increased by $\frac{1}{2}$. Find the number.
8. If the difference of a number and four is doubled, the result is $\frac{1}{4}$ less than the number. Find the number.

Solve. See Examples 3 through 5.

9. A 17-foot piece of string is cut into two pieces so that the longer piece is 2 feet longer than twice the shorter piece. Find the lengths of both pieces.

10. A 25-foot wire is to be cut so that the longer piece is one foot longer than 5 times the shorter piece. Find the length of each piece.

11. The largest meteorite in the world is the Hoba West located in Namibia. Its weight is 3 times the weight of the Armanty meteorite located in Outer Mongolia. If the sum of their weights is 88 tons, find the weight of each.

12. The area of the Sahara Desert is 7 times the area of the Gobi Desert. If the sum of their areas is 4,000,000 square miles, find the area of each desert.

13. The countries with the most cinema screens in the world are China and the United States. China has 5806 more cinema screens than the United States whereas the total screens for both countries is 78,994. Find the number of cinema screens for both countries. (Source: Film Distributor’s Association)

14. The countries with the most television stations in the world are Russia and China. Russia has 4066 more television stations than China whereas the total stations for both countries is 10,546. Find the number of television stations for both countries. (Source: Central Intelligence Agency, The World Factbook 2006)

15. The flag of Equatorial Guinea contains an isosceles triangle. (Recall that an isosceles triangle contains two angles with the
same measure.\) If the measure of the third angle of the triangle is 30° more than twice the measure of either of the other two angles, find the measure of each angle of the triangle. \(\text{Hint: Recall that the sum of the measures of the angles of a triangle is 180°.}\)

**Solve.** See Example 6. Fill in the table. Most of the first row has been completed for you.

<table>
<thead>
<tr>
<th>First Integer</th>
<th>Next Integers</th>
<th>Indicated Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integer: (x)</td>
<td>(x + 1)</td>
<td>(x + 2)</td>
</tr>
<tr>
<td>Integer: (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Even integer: (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Odd integer: (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integer: (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integer: (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Odd integer: (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Even integer: (x)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

25. The left and right page numbers of an open book are two consecutive integers whose sum is 469. Find these page numbers.

26. The room numbers of two adjacent classrooms are two consecutive even numbers. If their sum is 654, find the classroom numbers.

27. To make an international telephone call, you need the code for the country you are calling. The codes for Belgium, France, and Spain are three consecutive integers whose sum is 99. Find the code for each country. \(\text{Source: The World Almanac and Book of Facts, 2007}\)

28. To make an international telephone call, you need the code for the country you are calling. The codes for Mali Republic, Côte d’Ivoire, and Niger are three consecutive odd integers whose sum is 675. Find the code for each country.

**MIXED PRACTICE**

Solve. See Examples 1 through 6.

29. A 25-inch piece of steel is cut into three pieces so that the second piece is twice as long as the first piece, and the third piece is one inch more than five times the length of the first piece. Find the lengths of the pieces.
30. A 46-foot piece of rope is cut into three pieces so that the second piece is three times as long as the first piece, and the third piece is two feet more than seven times the length of the first piece. Find the lengths of the pieces.

31. Five times a number, subtracted from ten, is triple the number. Find the number.

32. Nine is equal to ten subtracted from double a number. Find the number.

33. The greatest producer of diamonds in carats is Botswana. This country produces about four times the amount produced in Angola. If the total produced in both countries is 40,000,000 carats, find the amount produced in each country. (Source: Diamond Facts 2006.)

34. Beetles have the greatest number of different species. There are twenty times the number of beetle species as grasshopper species, and the total number of species for both is 420,000. Find the number of species for each type of insect.

35. The measures of the angles of a triangle are 3 consecutive even integers. Find the measure of each angle.

36. A quadrilateral is a polygon with 4 sides. The sum of the measures of the 4 angles in a quadrilateral is 360°. If the measures of the angles of a quadrilateral are consecutive odd integers, find the measures.

37. For the 2006 Winter Olympics, the total number of medals won by athletes in each of the countries of Russia, Austria, Canada, and the United States are four consecutive integers whose sum is 94. Find the number of medals for each country.

38. The code to unlock a student's combination lock happens to be three consecutive odd integers whose sum is 51. Find the integers.

39. If the sum of a number and five is tripled, the result is one less than twice the number. Find the number.

40. Twice the sum of a number and six equals three times the sum of the number and four. Find the number.

41. In a recent election in Illinois for a seat in the United States House of Representatives, Jerry Weller received 20,196 more votes than opponent John Pavich. If the total number of votes was 196,554, find the number of votes for each candidate. (Source: The Washington Post)

42. In a recent election in New York for a seat in the United States House of Representatives, Timothy Bishop received 35,650 more votes than opponent Italo Zanzi. If the total number of votes was 158,192, find the number of votes for each candidate. (Source: The New York Times)

43. Two angles are supplementary if their sum is 180°. The larger angle measures eight degree more than three times the measure of a smaller angle. If \( x \) represents the measure of the smaller angle and these two angles are supplementary, find the measure of each angle.

44. Two angles are complementary if their sum is 90°. The larger angle measures three degrees less than twice the measure of a smaller angle. If \( x \) represents the measure of the smaller angle and these two angles are complementary, find the measure of each angle.

45. If the quotient of a number and 4 is added to \( \frac{1}{2} \), the result is \( \frac{3}{4} \). Find the number.

46. If \( \frac{3}{4} \) is added to three times a number, the result is \( \frac{1}{2} \) subtracted from twice the number. Find the number.

47. The flag of Brazil contains a parallelogram. One angle of the parallelogram is 15° less than twice the measure of the angle next to it. Find the measure of each angle of the parallelogram. (Hint: Recall that opposite angles of a parallelogram have the same measure and that the sum of the measures of the angles is 360°.)

48. The sum of the measures of the angles of a parallelogram is 360°. In the parallelogram below, angles \( A \) and \( D \) have the same measure as well as angles \( C \) and \( B \). If the measure of angle \( C \) is twice the measure of angle \( A \), find the measure of each angle.
49. Currently, the two fastest trains are the Japanese Maglev and the French TGV. The sum of their fastest speeds is 718.2 miles per hour. If the speed of the Maglev is 3.8 mph faster than the speed of the TGV, find the speeds of each.

50. The Pentagon is the world’s largest office building in terms of floor space. It has three times the amount of floor space as the Empire State Building. If the total floor space for these two buildings is approximately 8700 thousand square feet, find the floor space of each building.

51. One-third of a number is five-sixths. Find the number.

52. Seven-eighths of a number is one-half. Find the number.

53. The number of counties in California and the number of counties in Montana are consecutive even integers whose sum is 114. If California has more counties than Montana, how many counties does each state have? (Source: The World Almanac and Book of Facts 2007)

54. A student is building a bookcase with stepped shelves for her dorm room. She buys a 48-inch board and wants to cut the board into three pieces with lengths equal to three consecutive even integers. Find the three board lengths.

55. In Super Bowl XLI in Miami, Florida, the Indianapolis Colts won over the Chicago Bears with a 12-point lead. If the total of the two scores was 46, find the individual team scores. (Source: National Football League)

56. During the 2007 Rose Bowl, University of Southern California beat Michigan by 14 points. If their combined scores total 50, find the individual team scores. (Source: ESPN Sports Almanac)

57. A geodesic dome, based on the design by Buckminster Fuller, is composed of two different types of triangular panels. One of these is an isosceles triangle. In one geodesic dome, the measure of the third angle is 76.5° more than the measure of either of the two equal angles. Find the measure of the third angle. (Source: Buckminster Fuller Institute)

58. The measures of the angles of a particular triangle are such that the second and third angles are each four times larger than the smallest angle. Find the measures of the angles of this triangle.

59. A 40-inch board is to be cut into three pieces so that the second piece is twice as long as the first piece and the third piece is 5 times as long as the first piece. If \( x \) represents the length of the first piece, find the lengths of all three pieces.

60. A 30-foot piece of siding is cut into three pieces so that the second piece is four times as long as the first piece and the third piece is five times as long as the first piece. If \( x \) represents the length of the first piece, find the lengths of all three pieces.

The graph below shows the states with the highest tourism budgets. Use the graph for Exercises 61 through 66.

61. Which state spends the most money on tourism?

62. Which states spend between $30 and $40 million on tourism?
63. The states of Texas and Florida spend a total of $60.5 million for tourism. The state of Texas spends $1.7 million more than the state of Florida. Find the amount that each state spends on tourism.

64. The states of Hawaii and Pennsylvania spend a total of $91.1 million for tourism. The state of Hawaii spends $14.2 million less than twice the amount of money that the state of Pennsylvania spends. Find the amount that each state spends on tourism.

Compare the heights of the bars in the graph with your results of the exercises below. Are your answers reasonable?

65. Exercise 63
66. Exercise 64

2.5 FORMULAS AND PROBLEM SOLVING

OBJECTIVES

1. Use formulas to solve problems.
2. Solve a formula or equation for one of its variables.

OBJECTIVE 1 Using formulas to solve problems. An equation that describes a known relationship among quantities, such as distance, time, volume, weight, and money is called a formula. These quantities are represented by letters and are thus variables of the formula. Here are some common formulas and their meanings.

<table>
<thead>
<tr>
<th>Formulas and Their Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = lw$</td>
</tr>
<tr>
<td>$I = PRT$</td>
</tr>
<tr>
<td>$P = a + b + c$</td>
</tr>
<tr>
<td>$d = rt$</td>
</tr>
<tr>
<td>$V = lwh$</td>
</tr>
</tbody>
</table>

$F = \left(\frac{9}{5}\right)C + 32$ or $F = 1.8C + 32$

degrees Fahrenheit = \left(\frac{9}{5}\right) \cdot$degrees Celsius + 32

Formulas are valuable tools because they allow us to calculate measurements as long as we know certain other measurements. For example, if we know we traveled a distance of 100 miles at a rate of 40 miles per hour, we can replace the variables $d$ and $r$ in the formula $d = rt$ and find our time, $t$.

$d = rt$  \hspace{1cm} \text{Formula.}$
$100 = 40t$  \hspace{1cm} \text{Replace } d \text{ with 100 and } r \text{ with 40.}$
This is a linear equation in one variable, $t$. To solve for $t$, divide both sides of the equation by 40.

\[
\frac{100}{40} = \frac{40t}{40} \quad \text{Divide both sides by 40.}
\]

\[
\frac{5}{2} = t \quad \text{Simplify.}
\]

The time traveled is $\frac{5}{2}$ hours or $2 \frac{1}{2}$ hours.

In this section we solve problems that can be modeled by known formulas. We use the same problem-solving steps that were introduced in the previous section. These steps have been slightly revised to include formulas.

**Example 1** Finding Time Given Rate and Distance

A glacier is a giant mass of rocks and ice that flows downhill like a river. Portage Glacier in Alaska is about 6 miles, or 31,680 feet, long and moves 400 feet per year. Icebergs are created when the front end of the glacier flows into Portage Lake. How long does it take for ice at the head (beginning) of the glacier to reach the lake?

**Solution**

1. **UNDERSTAND.** Read and reread the problem. The appropriate formula needed to solve this problem is the distance formula, $d = rt$. To become familiar with this formula, let’s find the distance that ice traveling at a rate of 400 feet per year travels in 100 years. To do so, we let time $t$ be 100 years and rate $r$ be the given 400 feet per year, and substitute these values into the formula $d = rt$. We then have that distance $d = 400(100) = 40,000$ feet. Since we are interested in finding how long it takes ice to travel 31,680 feet, we now know that it is less than 100 years.

   Since we are using the formula $d = rt$, we let
   
   $t =$ the time in years for ice to reach the lake
   $r =$ rate or speed of ice
   $d =$ distance from beginning of glacier to lake

2. **TRANSLATE.** To translate to an equation, we use the formula $d = rt$ and let distance $d = 31,680$ feet and rate $r = 400$ feet per year.

\[
31,680 = 400 \cdot t \quad \text{Let } d = 31,680 \text{ and } r = 400.
\]
3. SOLVE. Solve the equation for \( t \). To solve for \( t \), divide both sides by 400.

\[
\frac{31,680}{400} = \frac{400 \cdot t}{400} = 79.2 = t
\]

Divide both sides by 400.

Simplify.

4. INTERPRET.

Check: To check, substitute 79.2 for \( t \) and 400 for \( r \) in the distance formula and check to see that the distance is 31,680 feet.

State: It takes 79.2 years for the ice at the head of Portage Glacier to reach the lake.

PRACTICE

The Stromboli Volcano, in Italy, began erupting in 2002, after a dormant period of over 17 years. In 2007, a vulcanologist measured the lava flow to be moving at 5 meters/second. If the path the lava followed to the sea is 580 meters long, how long does it take the lava to reach the sea? (Source: Thorsten Boeckel and CNN)

EXAMPLE 2 Calculating the Length of a Garden

Charles Pecot can afford enough fencing to enclose a rectangular garden with a perimeter of 140 feet. If the width of his garden must be 30 feet, find the length.

Solution

1. UNDERSTAND. Read and reread the problem. The formula needed to solve this problem is the formula for the perimeter of a rectangle, \( P = 2l + 2w \). Before continuing, let’s become familiar with this formula.

   \( l \) = the length of the rectangular garden
   \( w \) = the width of the rectangular garden
   \( P \) = perimeter of the garden

2. TRANSLATE. To translate to an equation, we use the formula \( P = 2l + 2w \) and let perimeter \( P = 140 \) feet and width \( w = 30 \) feet.

\[
P = 2l + 2w
\]

\[
140 = 2l + 2(30)
\]

Let \( P = 140 \) and \( w = 30 \).

3. SOLVE.

\[
140 = 2l + 2(30)
\]

\[
140 = 2l + 60
\]

Multiply 2(30).

\[
140 - 60 = 2l + 60 - 60
\]

Subtract 60 from both sides.

\[
80 = 2l
\]

Combine like terms.

\[
40 = l
\]

Divide both sides by 2.
Chapter 2  Equations, Inequalities, and Problem Solving

4. INTERPRET.
Check: Substitute 40 for \( l \) and 30 for \( w \) in the perimeter formula and check to see that the perimeter is 140 feet.

State: The length of the rectangular garden is 40 feet.

EXAMPLE 3  Finding an Equivalent Temperature

Evelyn Gryk fenced in part of her back yard for a dog run. The dog run was 40 feet in length and used 98 feet of fencing. Find the width of the dog run.

PrACTICE 2  Evelyn Gryk fenced in part of her back yard for a dog run. The dog run was 40 feet in length and used 98 feet of fencing. Find the width of the dog run.

EXAMPLE 3  Finding an Equivalent Temperature

The average minimum temperature for July in Shanghai, China, is 77° Fahrenheit. Find the equivalent temperature in degrees Celsius.

Solution

1. UNDERSTAND. Read and reread the problem. A formula that can be used to solve this problem is the formula for converting degrees Celsius to degrees Fahrenheit, \( F = \frac{9}{5}C + 32 \). Before continuing, become familiar with this formula. Using this formula, we let

\[
C = \text{temperature in degrees Celsius, and}
\]
\[
F = \text{temperature in degrees Fahrenheit.}
\]

2. TRANSLATE. To translate to an equation, we use the formula \( F = \frac{9}{5}C + 32 \) and let degrees Fahrenheit \( F = 77 \).

Formula: \( F = \frac{9}{5}C + 32 \)

Substitute: \( 77 = \frac{9}{5}C + 32 \)  Let \( F = 77 \).

3. SOLVE.

\[
77 = \frac{9}{5}C + 32
\]

\[
77 - 32 = \frac{9}{5}C + 32 - 32 \hspace{1cm} \text{Subtract 32 from both sides.}
\]

\[
45 = \frac{9}{5}C \hspace{1cm} \text{Combine like terms.}
\]

\[
5 \cdot 45 = 5 \cdot \frac{9}{5}C \hspace{1cm} \text{Multiply both sides by} \hspace{1cm} \frac{5}{5}
\]

\[
25 = C \hspace{1cm} \text{Simplify.}
\]

4. INTERPRET.

Check: To check, replace \( C \) with 25 and \( F \) with 77 in the formula and see that a true statement results.

State: Thus, 77° Fahrenheit is equivalent to 25° Celsius.

Note: There is a formula for directly converting degrees Fahrenheit to degrees Celsius. It is \( C = \frac{5}{9}(F - 32) \), as we shall see in Example 8.

PrACTICE 3  The average minimum temperature for July in Sydney, Australia, is 8° Celsius. Find the equivalent temperature in degrees Fahrenheit.
In the next example, we again use the formula for perimeter of a rectangle as in Example 2. In Example 2, we knew the width of the rectangle. In this example, both the length and width are unknown.

**EXAMPLE 4**  Finding Road Sign Dimensions

The length of a rectangular road sign is 2 feet less than three times its width. Find the dimensions if the perimeter is 28 feet.

**Solution**

1. **UNDERSTAND.** Read and reread the problem. Recall that the formula for the perimeter of a rectangle is \( P = 2l + 2w \). Draw a rectangle and guess the solution. If the width of the rectangular sign is 5 feet, its length is 2 feet less than 3 times the width or \( 3(5\text{ feet}) - 2 \text{ feet} = 13 \text{ feet} \). The perimeter \( P \) of this rectangle, drawn above, is then \( 2(13 \text{ feet}) + 2(5 \text{ feet}) = 36 \text{ feet} \), too much. We now know that the width is less than 5 feet.

Let \( w = \) the width of the rectangular sign; then \( 3w - 2 = \) the length of the sign.

Draw a rectangle and label it with the assigned variables, as shown in the left margin.

2. **TRANSLATE.**

- **Formula:** \( P = 2l + 2w \)
- **Substitute:** \( 28 = 2(3w - 2) + 2w \).

3. **SOLVE.**

\[
28 = 2(3w - 2) + 2w \\
28 = 6w - 4 + 2w \\
28 = 8w - 4 \\
28 + 4 = 8w - 4 + 4 \\
32 = 8w \\
32 \div 8 = 8w \div 8 \\
4 = w
\]

4. **INTERPRET.**

**Check:** If the width of the sign is 4 feet, the length of the sign is \( 3(4 \text{ feet}) - 2 \text{ feet} = 10 \text{ feet} \). This gives a perimeter of \( P = 2(4 \text{ feet}) + 2(10 \text{ feet}) = 28 \text{ feet} \), the correct perimeter.

**State:** The width of the sign is 4 feet and the length of the sign is 10 feet.

**PRACTICE**

The new street signs along Route 114 have a length that is 3 inches more than 5 times the width. Find the dimensions of the signs if the perimeter of the signs is 66 inches.
OBJECTIVE 2 Solving a formula for one of its variables. We say that the formula \( F = \frac{9}{5}C + 32 \) is solved for \( F \) because \( F \) is alone on one side of the equation and the other side of the equation contains no \( F \)'s. Suppose that we need to convert many Fahrenheit temperatures to equivalent degrees Celsius. In this case, it is easier to perform this task by solving the formula \( F = \frac{9}{5}C + 32 \) for \( C \). (See Example 8.) For this reason, it is important to be able to solve an equation for any one of its specified variables. For example, the formula \( d = rt \) is solved for \( d \) in terms of \( r \) and \( t \). We can also solve \( d = rt \) for \( t \) in terms of \( d \) and \( r \). To solve for \( t \), divide both sides of the equation by \( r \).

\[
\begin{align*}
\frac{d}{r} &= \frac{rt}{r} \\
\frac{d}{r} &= t \\
\end{align*}
\]

Divide both sides by \( r \).

Simplify.

To solve a formula or an equation for a specified variable, we use the same steps as for solving a linear equation. These steps are listed next.

**Solving Equations for a Specified Variable**

**STEP 1.** Multiply on both sides to clear the equation of fractions if they occur.

**STEP 2.** Use the distributive property to remove parentheses if they occur.

**STEP 3.** Simplify each side of the equation by combining like terms.

**STEP 4.** Get all terms containing the specified variable on one side and all other terms on the other side by using the addition property of equality.

**STEP 5.** Get the specified variable alone by using the multiplication property of equality.

**EXAMPLE 5** Solve \( V = lwh \) for \( l \).

**Solution** This formula is used to find the volume of a box. To solve for \( l \), divide both sides by \( wh \).

\[
\begin{align*}
\frac{V}{wh} &= \frac{lwh}{wh} \\
\frac{V}{wh} &= l \\
\end{align*}
\]

Divide both sides by \( wh \).

Simplify.

Since we have \( l \) alone on one side of the equation, we have solved for \( l \) in terms of \( V \), \( w \), and \( h \). Remember that it does not matter on which side of the equation we isolate the variable.

**PRACTICE** Solve \( I = Prt \) for \( r \).

**EXAMPLE 6** Solve \( y = mx + b \) for \( x \).

**Solution** The term containing the variable we are solving for, \( mx \), is on the right side of the equation. Get \( mx \) alone by subtracting \( b \) from both sides.

\[
\begin{align*}
y &= mx + b \\
y - b &= mx + b - b \\
y - b &= mx \\
\end{align*}
\]

Subtract \( b \) from both sides.

Combine like terms.
Next, solve for \( x \) by dividing both sides by \( m \).

\[
\frac{y - b}{m} = \frac{mx}{m} \\
\frac{y - b}{m} = x
\]

Simplify.

Solve for \( s \).

\[
H = 5s + 10 \quad \text{for} \quad s
\]

\[
S = 5s + 10 \\
S = 5s + 10 - 10 \\
S = 5s
\]

Multiply both sides by \( 9 \).

\[
9S = 9(5s) + 9(10) \\
9S = 45s + 90
\]

Divide both sides by \( 9 \).

\[
\frac{9S}{9} = \frac{45s + 90}{9} \\
S = 5s + 10
\]

Simplify.

\[
S = 5s + 10 \\
S = 5s + 10 - 10 \\
S = 5s
\]

The next example has an equation containing a fraction. We will first clear the equation of fractions and then solve for the specified variable.

**EXAMPLE 8** Solve \( F = \frac{9}{5}C + 32 \) for \( C \).

**Solution**

\[
F = \frac{9}{5}C + 32 \\
5F = 9C + 160 \\
5F - 160 = 9C \\
\frac{5F - 160}{9} = C
\]

To get the term containing the variable \( C \) alone, subtract 160 from both sides.

Divide both sides by 9.

\[
C = \frac{5}{9}(F - 32)
\]

Another equivalent way to write this formula is \( C = \frac{5}{9}(F - 32) \).

**PRACTICE 8** Solve \( A = \frac{1}{2}a(b + B) \) for \( B \).
118 CHAPTER 2 Equations, Inequalities, and Problem Solving

2.5 EXERCISE SET

Substitute the given values into each given formula and solve for the unknown variable. If necessary, round to one decimal place. See Examples 1 through 3.

1. \( A = bh; \quad A = 45, b = 15 \) (Area of a parallelogram)
2. \( d = rt; \quad d = 195, t = 3 \) (Distance formula)
3. \( S = 4lw + 2wh; \quad S = 102, l = 7, w = 3 \) (Surface area of a special rectangular box)
4. \( V = lwh; \quad l = 14, w = 8, h = 3 \) (Volume of a rectangular box)
5. \( A = \frac{1}{2}h(B + b); \quad A = 180, B = 11, b = 7 \) (Area of a trapezoid)
6. \( A = \frac{1}{2}h(B + b); \quad A = 60, B = 7, b = 3 \) (Area of a trapezoid)
7. \( P = a + b + c; \quad P = 30, a = 8, b = 10 \) (Perimeter of a triangle)
8. \( V = \frac{1}{3}Ah; \quad V = 45, h = 5 \) (Volume of a pyramid)
9. \( C = 2\pi r; \quad C = 15.7 \) (use the approximation 3.14 or a calculator approximation for \( \pi \)) (Circumference of a circle)
10. \( A = \pi r^2; \quad r = 4.5 \) (use the approximation 3.14 or a calculator approximation for \( \pi \)) (Area of a circle)
11. \( I = PRT; \quad I = 3750, P = 25,000, R = 0.05 \) (Simple interest formula)
12. \( I = PRT; \quad I = 1,056,000, P = 25,000, R = 0.05 \) (Simple interest formula)
13. \( V = \frac{1}{3}\pi r^2h; \quad V = 565.2, r = 6 \) (use a calculator approximation for \( \pi \)) (Volume of a cone)
14. \( V = \frac{4}{3}\pi r^3; \quad r = 3 \) (use a calculator approximation for \( \pi \)) (Volume of a sphere)

Solve each formula for the specified variable. See Examples 5 through 8.

15. \( f = 5gh \) for \( h \)
16. \( A = \pi ab \) for \( b \)
17. \( V = lwh \) for \( w \)
18. \( T = mn \) for \( n \)
19. \( 3x + y = 7 \) for \( y \)
20. \(-x + y = 13 \) for \( y \)
21. \( A = P + PRT \) for \( R \)
22. \( A = P + PRT \) for \( T \)
23. \( V = \frac{1}{3}Ah \) for \( A \)
24. \( D = \frac{1}{4}l \) for \( k \)
25. \( P = a + b + c \) for \( a \)
26. \( PR = x + y + z + w \) for \( z \)
27. \( S = 2\pi rh + 2\pi r^2 \) for \( h \)
28. \( S = 4lw + 2wh \) for \( h \)

29. For the purpose of purchasing new baseboard and carpet,
   a. Find the area and perimeter of the room below (neglecting doors).
   b. Identify whether baseboard has to do with area or perimeter and the same with carpet.

30. For the purpose of purchasing lumber for a new fence and seed to plant grass,
   a. Find the area and perimeter of the yard below.
   b. Identify whether a fence has to do with area or perimeter and the same with grass seed.

31. A frame shop charges according to both the amount of framing needed to surround the picture and the amount of glass needed to cover the picture.
   a. Find the area and perimeter of the trapezoid-shaped framed picture below.
   b. Identify whether the amount of framing has to do with perimeter or area and the same with the amount of glass.

32. A decorator is painting and placing a border completely around the parallelogram-shaped wall.
   a. Find the area and perimeter of the wall below.
   b. Identify whether the border has to do with perimeter or area and the same with paint.
33. The world’s largest pink ribbon, the sign of the fight against breast cancer, was erected out of pink post-it notes on a billboard in New York City in October, 2004. If the area of the rectangular billboard covered by the ribbon is approximately 3990 square feet, and the width of the billboard was approximately 57 feet, what was the height of this billboard?

34. The world’s largest sign for Coca-Cola is located in Arica, Chile. The rectangular sign has a length of 400 feet and has an area of 52,400 square feet. Find the width of the sign. (Source: Fabulous Facts about Coca-Cola, Atlanta, GA)

35. Convert Nome, Alaska’s 14°F high temperature to Celsius.

36. Convert Paris, France’s low temperature of −5°C to Fahrenheit.

37. The X-30 is a “space plane” that skims the edge of space at 4000 miles per hour. Neglecting altitude, if the circumference of the Earth is approximately 25,000 miles, how long will it take for the X-30 to travel around the Earth?

38. In the United States, a notable hang glider flight was a 303-mile, 8 1/2 hour flight from New Mexico to Kansas. What was the average rate during this flight?

39. An architect designs a rectangular flower garden such that the width is exactly two-thirds of the length. If 260 feet of antique picket fencing are to be used to enclose the garden, find the dimensions of the garden.

40. If the length of a rectangular parking lot is 10 meters less than twice its width, and the perimeter is 400 meters, find the length of the parking lot.

41. A flower bed is in the shape of a triangle with one side twice the length of the shortest side, and the third side is 30 feet more than the length of the shortest side. Find the dimensions if the perimeter is 102 feet.

42. The perimeter of a yield sign in the shape of an isosceles triangle is 22 feet. If the shortest side is 2 feet less than the other two sides, find the length of the shortest side. (Hint: An isosceles triangle has two sides the same length.)

43. The Cat is a high-speed catamaran auto ferry that operates between Bar Harbor, Maine, and Yarmouth, Nova Scotia. The Cat can make the 138-mile trip in about 1 1/2 hours. Find the catamaran speed for this trip. (Source: Bay Ferries)

44. A family is planning their vacation to Disney World. They will drive from a small town outside New Orleans, Louisiana, to Orlando, Florida, a distance of 700 miles. They plan to average a rate of 55 mph. How long will this trip take?

45. Piranha fish require 1.5 cubic feet of water per fish to maintain a healthy environment. Find the maximum number of piranhas you could put in a tank measuring 8 feet by 3 feet by 6 feet.

46. Find the maximum number of goldfish you can put in a cylindrical tank whose diameter is 8 meters and whose height is 3 meters if each goldfish needs 2 cubic meters of water.

47. A lawn is in the shape of a trapezoid with a height of 60 feet and bases of 70 feet and 130 feet. How many whole bags of fertilizer must be purchased to cover the lawn if each bag covers 4000 square feet?
48. If the area of a right-triangly shaped sail is 20 square feet and its base is 5 feet, find the height of the sail.

49. Maria’s Pizza sells one 16-inch cheese pizza or two 10-inch cheese pizzas for $9.99. Determine which size gives more pizza.

50. Find how much rope is needed to wrap around the Earth at the equator, if the radius of the Earth is 4000 miles. (Hint: Use 3.14 for $\pi$ and the formula for circumference.)

51. The perimeter of a geometric figure is the sum of the lengths of its sides. If the perimeter of the following pentagon (five-sided figure) is 48 meters, find the length of each side.

52. The perimeter of the following triangle is 82 feet. Find the length of each side.

53. A Japanese “bullet” train set a new world record for train speed at 361 miles per hour during a manned test run on the Yamanashi Maglev Test Line in 2003. How long does it take this train to travel 72.2 miles at this speed? Give the result in hours; then convert to minutes.

54. In 1983, the Hawaiian volcano Kilauea began erupting in a series of episodes still occurring at the time of this writing. At times, the lava flows advanced at speeds of up to 0.5 kilometer per hour. In 1983 and 1984 lava flows destroyed 16 homes in the Royal Gardens subdivision, about 6 km away from the eruption site. Roughly how long did it take the lava to reach Royal Gardens? Assume that the lava traveled at its fastest rate, 0.5 kph. (Source: U.S. Geological Survey Hawaiian Volcano Observatory)

55. The perimeter of an equilateral triangle is 7 inches more than the perimeter of a square, and the side of the triangle is 5 inches longer than the side of the square. Find the side of the triangle. (Hint: An equilateral triangle has three sides the same length.)

56. A square animal pen and a pen shaped like an equilateral triangle have equal perimeters. Find the length of the sides of each pen if the sides of the triangular pen are fifteen less than twice a side of the square pen.

57. Find how long it takes a person to drive 135 miles on I-10 if she merges onto I-10 at 10 a.m. and drives nonstop with her cruise control set on 60 mph.

58. Beaumont, Texas, is about 150 miles from Toledo Bend. If Leo Miller leaves Beaumont at 4 a.m. and averages 45 mph, when should he arrive at Toledo Bend?

59. The longest runway at Los Angeles International Airport has the shape of a rectangle and an area of 1,813,500 square feet. This runway is 150 feet wide. How long is the runway? (Source: Los Angeles World Airports)

60. Normal room temperature is about 78°F. Convert this temperature to Celsius.

61. The highest temperature ever recorded in Europe was 122°F in Seville, Spain, in August of 1881. Convert this record high temperature to Celsius. (Source: National Climatic Data Center)

62. The lowest temperature ever recorded in Oceania was −10°C at the Haleakala Summit in Maui, Hawaii, in January 1961. Convert this record low temperature to Fahrenheit. (Source: National Climatic Data Center)

63. The CART FedEx Championship Series is an open-wheeled race car competition based in the United States. A CART car has a maximum length of 199 inches, a maximum width of 78.5 inches, and a maximum height of 33 inches. When the CART series travels to another country for a grand prix, teams must ship their cars. Find the volume of the smallest
64. On a road course, a CART car's speed can average up to around 105 mph. Based on this speed, how long would it take a CART driver to travel from Los Angeles to New York City, a distance of about 2810 miles by road, without stopping? Round to the nearest tenth of an hour.

65. The Hoberman Sphere is a toy ball that expands and contracts. When it is completely closed, it has a diameter of 9.5 inches. Find the volume of the Hoberman Sphere when it is completely closed. Use 3.14 for . Round to the nearest whole cubic inch. (Source: Hoberman Designs, Inc.)

66. When the Hoberman Sphere (see Exercise 65) is completely expanded, its diameter is 30 inches. Find the volume of the Hoberman Sphere when it is completely expanded. Use 3.14 for . Round to the nearest whole cubic inch. (Source: Hoberman Designs, Inc.)

67. The average temperature on the planet Mercury is 167°C. Convert this temperature to degrees Fahrenheit. (Source: National Space Science Data Center)

68. The average temperature on the planet Jupiter is −227°F. Convert this temperature to degrees Celsius. Round to the nearest degree. (Source: National Space Science Data Center)

**REVIEW AND PREVIEW**

Write the following phrases as algebraic expressions. See Section 2.1.

69. Nine divided by the sum of a number and 5

70. Half the product of a number and five

71. Three times the sum of a number and four

72. Double the sum of ten and four times a number

73. Triple the difference of a number and twelve

74. A number minus the sum of the number and six

**CONCEPT EXTENSIONS**

Solve. See the Concept Check in this section.

75. for

76. for

77. Dry ice is a name given to solidified carbon dioxide. At −78.5°C Celsius it changes directly from a solid to a gas. Convert this temperature to Fahrenheit.

78. Lightning bolts can reach a temperature of 50,000°F Fahrenheit. Convert this temperature to Celsius.

79. The distance from the sun to the Earth is approximately 93,000,000 miles. If light travels at a rate of 186,000 miles per second, how long does it take light from the sun to reach us?

80. Light travels at a rate of 186,000 miles per second. If our moon is 238,860 miles from the Earth, how long does it take light from the moon to reach us? (Round to the nearest tenth of a second.)

81. A glacier is a giant mass of rocks and ice that flows downhill like a river. Exit Glacier, near Seward, Alaska, moves at a rate of 20 inches a day. Find the distance in feet the glacier moves in a year. (Assume 365 days a year. Round to 2 decimal places.)

82. Flying fish do not actually fly, but glide. They have been known to travel a distance of 1300 feet at a rate of 20 miles per hour. How many seconds did it take to travel this distance? (Hint: First convert miles per hour to feet per second. Recall that 1 mile = 5280 feet. Round to the nearest tenth of a second.)
83. Stalactites join stalagmites to form columns. A column found at Natural Bridge Caverns near San Antonio, Texas, rises 15 feet and has a diameter of only 2 inches. Find the volume of this column in cubic inches. (Hint: Use the formula for volume of a cylinder and use a calculator approximation for \( \pi \). Round to the nearest tenth of an inch.)

84. Find the temperature at which the Celsius measurement and Fahrenheit measurement are the same number.

85. The formula \( A = bh \) is used to find the area of a parallelogram. If the base of a parallelogram is doubled and its height is doubled, how does this affect the area?

86. The formula \( V = LWH \) is used to find the volume of a box. If the length of a box is doubled, the width is doubled, and the height is doubled, how does this affect the volume?

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**Organizing a Notebook**

It’s never too late to get organized. If you need ideas about organizing a notebook for your mathematics course, try some of these:

- Use a spiral or ring binder notebook with pockets and use it for mathematics only.
- Start each page by writing the book’s section number you are working on at the top.
- When your instructor is lecturing, take notes. Always include any examples your instructor works for you.
- Place your worked-out homework exercises in your notebook immediately after the lecture notes from that section. This way, a section’s worth of material is together.
- Homework exercises: Attempt and check all assigned homework.
- Place graded quizzes in the pockets of your notebook or a special section of your binder.

**Self-Check**

Check your notebook organization by answering the following questions.

1. Do you have a spiral or ring binder notebook for your mathematics course only?
2. Have you ever had to flip through several sheets of notes and work in your mathematics notebook to determine what section’s work you are in?
3. Are you now writing the textbook’s section number at the top of each notebook page?
4. Have you ever lost or had trouble finding a graded quiz or test?
5. Are you now placing all your graded work in a dedicated place in your notebook?
6. Are you attempting all of your homework and placing all of your work in your notebook?
7. Are you checking and correcting your homework in your notebook? If not, why not?
8. Are you writing in your notebook the examples your instructor works for you in class?
This section is devoted to solving problems in the categories listed. The same problem-solving steps used in previous sections are also followed in this section. They are listed below for review.

**OBJECTIVES**
1. Solve percent equations.
2. Solve discount and mark-up problems.
3. Solve percent increase and percent decrease problems.
4. Solve mixture problems.

**OBJECTIVE 1** Solving percent equations. Many of today’s statistics are given in terms of percent: a basketball player’s free throw percent, current interest rates, stock market trends, and nutrition labeling, just to name a few. In this section, we first explore percent, percent equations, and applications involving percents. See Appendix B.2 if a further review of percents is needed.

**EXAMPLE 1** The number 63 is what percent of 72?

**Solution**
1. **UNDERSTAND.** Read and reread the problem. Next, let’s suppose that the percent is 80%. To check, we find 80% of 72.

   \[ 80\% \text{ of } 72 = 0.80(72) = 57.6 \]

   This is close, but not 63. At this point, though, we have a better understanding of the problem, we know the correct answer is close to and greater than 80%, and we know how to check our proposed solution later.

   Let \( x \) = the unknown percent.

2. **TRANSLATE.** Recall that “is” means “equals” and “of” signifies multiplying. Let’s translate the sentence directly.

   \[
   \begin{align*}
   \text{the number 63} & \quad \text{is} & \quad \text{what percent} & \quad \text{of} & \quad 72 \\
   63 & = & x & \cdot & 72
   \end{align*}
   \]

3. **SOLVE.**

   \[
   \begin{align*}
   63 & = 72x \\
   0.875 & = x \quad \text{Divide both sides by 72.} \\
   87.5\% & = x \quad \text{Write as a percent.}
   \end{align*}
   \]
4. INTERPRET.

Check: Verify that 87.5% of 72 is 63.

State: The number 63 is 87.5% of 72.

EXAMPLE 2 The number 120 is 15% of what number?

Solution

1. UNDERSTAND. Read and reread the problem.

Let \( x \) = the unknown number.

2. TRANSLATE.

\[
\begin{array}{cccc}
\text{the number 120} & \text{is} & \text{15\%} & \text{of} & \text{what number} \\
120 & = & 15\% & \cdot & x \\
\end{array}
\]

3. SOLVE.

\[120 = 0.15x \quad \text{Write 15\% as 0.15.}
\]

\[800 = x \quad \text{Divide both sides by 0.15.}
\]

4. INTERPRET.

Check: Check the proposed solution by finding 15% of 800 and verifying that the result is 120.

State: Thus, 120 is 15% of 800.

EXAMPLE 3 The circle graph below shows the purpose of trips made by American travelers. Use this graph to answer the following questions.

The next example contains a circle graph. This particular circle graph shows percents of American travelers in certain categories. Since the circle graph represents all American travelers, the percents should add to 100%.

Helpful Hint
The percents in a circle graph should have a sum of 100%.

EXAMPLE 3 The circle graph below shows the purpose of trips made by American travelers. Use this graph to answer the following questions.

Purpose of American Travelers

- Personal/Other, 13%
- Business, 17%
- Pleasure, 66%
- Combined Business/Pleasure, 4%

Source: Travel Industry Association of America
Section 2.6 Percent and Mixture Problem Solving

a. What percent of trips made by American travelers are solely for the purpose of business?

b. What percent of trips made by American travelers are for the purpose of business or combined business/pleasure?

c. On an airplane flight of 253 Americans, how many of these people might we expect to be traveling solely for business?

**Solution**

a. From the circle graph, we see that 17% of trips made by American travelers are solely for the purpose of business.

b. From the circle graph, we know that 17% of trips are solely for business and 4% of trips are for combined businesses/pleasure. The sum 17% + 4% or 21% of trips made by American travelers are for the purpose of business or combined business/pleasure.

c. Since 17% of trips made by American travelers are for business, we find 17% of 253. Remember that “of” translates to “multiplication.”

\[
17\%\text{ of } 253 = 0.17(253) \quad \text{Replace “of” with the operation of multiplication.}
\]

\[
= 43.01
\]

We might then expect that about 43 American travelers on the flight are traveling solely for business.

**OBJECTIVE 2 ▶ Solving discount and mark-up problems.** The next example has to do with discounting the price of a cell phone.

**Example 4** Cell Phones Unlimited recently reduced the price of a $140 phone by 20%. What is the discount and the new price?

**Solution**

1. **UNDERSTAND.** Read and reread the problem. Make sure you understand the meaning of the word “discount.” Discount is the amount of money by which the cost of an item has been decreased. To find the discount, we simply find 20% of $140. In other words, we have the formulas,

\[
\text{discount} = \text{percent} \cdot \text{original price} \quad \text{Then}
\]

\[
\text{new price} = \text{original price} - \text{discount}
\]
2, 3. TRANSLATE and SOLVE.

\[
\text{discount} = \text{percent} \cdot \text{original price}
\]

\[
= 20\% \cdot 140 = 0.20 \cdot 140 = 28
\]

Thus, the discount in price is $28.

\[
\text{new price} = \text{original price} - \text{discount}
\]

\[
= 140 - 28 = 112
\]

4. INTERPRET.

Check: Check your calculations in the formulas, and also see if our results are reasonable. They are.

State: The discount in price is $28 and the new price is $112.

A used treadmill, originally purchased for $480, was sold at a garage sale at a discount of 85% of the original price. What was the discount and the new price?

A concept similar to discount is mark-up. What is the difference between the two? A discount is subtracted from the original price while a mark-up is added to the original price. For mark-ups,

\[
\text{mark-up} = \text{percent} \cdot \text{original price}
\]

\[
\text{new price} = \text{original price} + \text{mark-up}
\]

Mark-up exercises can be found in Exercise Set 2.7.

OBJECTIVE 3 Solving percent increase and percent decrease problems. Percent increase or percent decrease is a common way to describe how some measurement has increased or decreased. For example, crime increased by 8%, teachers received a 5.5% increase in salary, or a company decreased its employees by 10%. The next example is a review of percent increase.

EXAMPLE 5 The cost of attending a public college rose from $9258 in 1996 to $12,796 in 2006. Find the percent increase, rounded to the nearest tenth of a percent. (Source: The College Board)

Solution

1. UNDERSTAND. Read and reread the problem. Let's guess that the percent increase is 20%. To see if this is the case, we find 20% of $9258 to find the increase in cost. Then we add this increase to $9258 to find the new cost. In other words, 20% ($9258) = 0.20($9258) = $1851.60, the increase in cost. The new cost then would be $9258 + $1851.60 = $11,109.60, less than the actual new cost of $12,976. We now know that the increase is greater than 20% and we know how to check our proposed solution.

Let \(x\) = the percent increase.
2. TRANSLATE. First, find the increase, and then the percent increase. The increase in cost is found by

In words: \[ \text{increase} = \text{new cost} - \text{old cost} \]

Translate: \[ \text{increase} = 12,796 - 9258 = 3538 \]

Next, find the percent increase. The percent increase or percent decrease is always a percent of the original number or, in this case, the old cost.

In words: \[ \text{increase is what percent increase of old cost} \]

Translate: \[ 3538 = x \cdot 9258 \]

3. SOLVE.

\[ 3538 = x \cdot 9258 \quad \text{Divide both sides by 9258.} \]
\[ 0.382 \approx x \quad \text{Round to 3 decimal places.} \]
\[ 38.2\% \approx x \quad \text{Write as a percent.} \]

4. INTERPRET.

Check: Check the proposed solution, as shown in Step 1.

State: The percent increase in cost is approximately 38.2%.

PRACTICE 38.2% \[ 0.382 \approx x \]

Percent decrease is found using a similar method. First find the decrease, then determine what percent of the original or first amount is that decrease.

Read the next example carefully. For Example 5, we were asked to find percent increase. In Example 6, we are given the percent increase and asked to find the number before the increase.

EXAMPLE 6 Most of the movie screens globally project analog film, but the number of cinemas using digital is increasing. Find the number of digital screens worldwide last year if, after a 153% increase the number this year is 849. Round to the nearest whole number. (Source: Motion Picture Association of America)

Solution

1. UNDERSTAND. Read and reread the problem. Let’s guess a solution and see how we would check our guess. If the number of digital screens worldwide last year was 400, we would see if 400 plus the increase is 849; that is,

\[ 400 + 153\% (400) = 400 + 1.53(400) = 2.53(400) = 1012 \]

Since 1012 is too large, we know that our guess of 400 is too large. We also have a better understanding of the problem. Let

\[ x = \text{number of digital screens last year} \]
2. **TRANSLATE.** To translate an equation, we remember that

<table>
<thead>
<tr>
<th>In words:</th>
<th>number of digital screens last year</th>
<th>plus</th>
<th>increase</th>
<th>equals</th>
<th>number of digital screens this year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translate:</td>
<td>( x )</td>
<td>+</td>
<td>1.53( x )</td>
<td>=</td>
<td>849</td>
</tr>
</tbody>
</table>

3. **SOLVE.**

\[
2.53x = 849 \quad \text{Add like terms.}
\]

\[
x = \frac{849}{2.53} = 336
\]

4. **INTERPRET.**

**Check:** Recall that \( x \) represents the number of digital screens worldwide last year. If this number is approximately 336, let’s see if 336 plus the increase is close to 849. (We use the word “close” since 336 is rounded.)

\[
336 + 153\% (336) = 336 + 1.53(336) = 2.53(336) = 850.08
\]

which is close to 849.

**State:** There were approximately 336 digital screens worldwide last year.

**PRACTICE 6** In 2005, 535 new feature films were released in the United States. This was an increase of 2.8% over the number of new feature films released in 2004. Find the number of new feature films released in 2004. (Source: Motion Picture Association of America)

**OBJECTIVE 4** Solving mixture problems. Mixture problems involve two or more different quantities being combined to form a new mixture. These applications range from Dow Chemical’s need to form a chemical mixture of a required strength to Planter’s Peanut Company’s need to find the correct mixture of peanuts and cashews, given taste and price constraints.

**EXAMPLE 7** Calculating Percent for a Lab Experiment

A chemist working on his doctoral degree at Massachusetts Institute of Technology needs 12 liters of a 50% acid solution for a lab experiment. The stockroom has only 40% and 70% solutions. How much of each solution should be mixed together to form 12 liters of a 50% solution?

**Solution:**

1. **UNDERSTAND.** First, read and reread the problem a few times. Next, guess a solution. Suppose that we need 7 liters of the 40% solution. Then we need \( 12 - 7 = 5 \) liters of the 70% solution. To see if this is indeed the solution, find the amount of pure acid in 7 liters of the 40% solution, in 5 liters of the 70% solution, and in 12 liters of a 50% solution, the required amount and strength.

\[
\begin{array}{ccc}
\text{number of liters} & \times & \text{acid strength} \\
7 \text{ liters} & \times & 40\% \\
5 \text{ liters} & \times & 70\% \\
12 \text{ liters} & \times & 50\%
\end{array}
\]

\[
\begin{array}{ccc}
\text{amount of pure acid} \\
7(0.40) \text{ or } 2.8 \text{ liters} \\
5(0.70) \text{ or } 3.5 \text{ liters} \\
12(0.50) \text{ or } 6 \text{ liters}
\end{array}
\]
Since 2.8 liters + 3.5 liters = 6.3 liters and not 6, our guess is incorrect, but we have gained some valuable insight into how to model and check this problem.

Let

\[ x = \text{number of liters of 40\% solution}; \text{ then} \]
\[ 12 - x = \text{number of liters of 70\% solution}. \]

2. TRANSLATE. To help us translate to an equation, the following table summarizes the information given. Recall that the amount of acid in each solution is found by multiplying the acid strength of each solution by the number of liters.

<table>
<thead>
<tr>
<th>No. of Liters</th>
<th>Acid Strength</th>
<th>Amount of Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% Solution</td>
<td>( x )</td>
<td>0.40</td>
</tr>
<tr>
<td>70% Solution</td>
<td>12 - x</td>
<td>0.70</td>
</tr>
<tr>
<td>50% Solution Needed</td>
<td>12</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The amount of acid in the final solution is the sum of the amounts of acid in the two beginning solutions.

In words: \[ \text{acid in 40\% solution} + \text{acid in 70\% solution} = \text{acid in 50\% mixture} \]

Translate: \[ 0.40x + 0.70(12 - x) = 0.50(12) \]

3. SOLVE.

Apply the distributive property.

\[ 0.4x + 8.4 - 0.7x = 6 \]
\[ -0.3x + 8.4 = 6 \]
Combine like terms.
\[ -0.3x = -2.4 \]
Subtract 8.4 from both sides.
\[ x = 8 \]
Divide both sides by \(-0.3\).

4. INTERPRET.

Check: To check, recall how we checked our guess.

State: If 8 liters of the 40\% solution are mixed with 12 - 8 or 4 liters of the 70\% solution, the result is 12 liters of a 50\% solution.

---

**VOCABULARY & READINESS CHECK**

Tell whether the percent labels in the circle graphs are correct.

1. 25\% 40\% 25\%
2. 30\% 30\% 30\%
3. 25\% 25\% 25\%
4. 40\% 50\% 10\%
Find each number described. See Examples 1 and 2.

1. What number is 16% of 70?
2. What number is 88% of 1000?
3. The number 28.6 is what percent of 52?
4. The number 87.2 is what percent of 436?
5. The number 45 is 25% of what number?
6. The number 126 is 35% of what number?

The circle graph below shows the uses of U.S. corn production. Use this graph for Exercises 7 through 10. See Example 3.

U.S. Corn Production Use
- Animal feed: 51%
- Exports: 19%
- Ethanol: 18%
- Food, Seed, Other: 12%

Source: USDA, American Farm Bureau Federation

7. What percent of corn production is used for animal feed or ethanol?
8. What percent of corn production is not used for exports?
9. The U.S. corn production in 2006–2007 was 10,535 million bushels. How many bushels were used to make ethanol?
10. How many bushels of the 2006–2007 corn production was used for food, seed, or other? (See Exercise 9.)

Solve. See Example 4.

11. A used automobile dealership recently reduced the price of a used compact car by 8%. If the price of the car before discount was $18,500, find the discount and the new price.
12. A music store is advertising a 25%-off sale on all new releases. Find the discount and the sale price of a newly released CD that regularly sells for $12.50.
13. A birthday celebration meal is $40.50 including tax. Find the total cost if a 15% tip is added to the cost.
14. A retirement dinner for two is $65.40 including tax. Find the total cost if a 20% tip is added to the cost.

Solve. See Example 5.

16. The cost of attending a private college rose from $19,000 in 2000 to $22,200 in 2006. Find the percent increase. Round to the nearest whole percent.

17. By decreasing each dimension by 1 unit, the area of a rectangle decreased from 40 square feet (on the left) to 28 square feet (on the right). Find the percent decrease in area.

18. By decreasing the length of the side by one unit, the area of a square decreased from 100 square meters to 81 square meters. Find the percent decrease in area.

Solve. See Example 6.

19. Find the original price of a pair of shoes if the sale price is $78 after a 25% discount.
20. Find the original price of a popular pair of shoes if the increased price is $80 after a 25% increase.
21. Find last year’s salary if after a 4% pay raise, this year’s salary is $44,200.
22. Find last year’s salary if after a 3% pay raise, this year’s salary is $55,620.

Solve. For each exercise, a table is given for you to complete and use to write an equation that models the situation. See Example 7.

23. How much pure acid should be mixed with 2 gallons of a 40% acid solution in order to get a 70% acid solution?

<table>
<thead>
<tr>
<th>Number of Acid Amount</th>
<th>Strength of Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Acid</td>
<td>100%</td>
</tr>
<tr>
<td>40% Acid Solution</td>
<td></td>
</tr>
<tr>
<td>70% Acid Solution Needed</td>
<td></td>
</tr>
</tbody>
</table>

24. How many cubic centimeters (cc) of a 25% antibiotic solution should be added to 10 cubic centimeters of a 60% antibiotic solution in order to get a 30% antibiotic solution?

<table>
<thead>
<tr>
<th>Number of Cubic cm</th>
<th>Antibiotic Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% Antibiotic Solution</td>
<td></td>
</tr>
<tr>
<td>60% Antibiotic Solution</td>
<td></td>
</tr>
<tr>
<td>30% Antibiotic Solution Needed</td>
<td></td>
</tr>
</tbody>
</table>
25. Community Coffee Company wants a new flavor of Cajun coffee. How many pounds of coffee worth $7 a pound should be added to 14 pounds of coffee worth $4 a pound to get a mixture worth $5 a pound?

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Cost per Pound</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7 per lb Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4 per lb Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 per lb Coffee Wanted</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

26. Planter’s Peanut Company wants to mix 20 pounds of peanuts worth $3 a pound with cashews worth $5 a pound in order to make an experimental mix worth $3.50 a pound. How many pounds of cashews should be added to the peanuts?

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Cost per Pound</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 per lb Peanuts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 per lb Cashews</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.50 per lb Mixture Wanted</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MIXED PRACTICE
Solve. If needed, round money amounts to two decimal places and all other amounts to one decimal place. See Examples 1 through 7.

27. Find 23% of 20.

28. Find 140% of 86.

29. The number 40 is 80% of what number?

30. The number 56.25 is 45% of what number?

31. The number 144 is what percent of 480?

32. The number 42 is what percent of 35?

33. Estimate the percent of the population in Fairbanks, Alaska, who shops by catalog.

34. Estimate the percent of the population in Charlottesville, Virginia, who shops by catalog.

35. According to the World Almanac, Anchorage has a population of 275,043. How many catalog shoppers might we predict live in Anchorage? Round to the nearest whole number.


For Exercises 37 and 38, fill in the percent column in each table. Each table contains a worked-out example.

37. Ford Motor Company
Model Year 2006 Vehicle Sales Worldwide

<table>
<thead>
<tr>
<th>Region</th>
<th>Thousands of Vehicles</th>
<th>Percent of Total (Rounded to Nearest Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>North America</td>
<td>3051</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>1846</td>
<td></td>
</tr>
<tr>
<td>Asia-Pacific-Africa</td>
<td>589</td>
<td></td>
</tr>
<tr>
<td>South America</td>
<td>381</td>
<td></td>
</tr>
<tr>
<td>Rest of the World</td>
<td>730</td>
<td>Example: ( \frac{730}{6597} \approx 11% )</td>
</tr>
<tr>
<td>Total</td>
<td>6597</td>
<td></td>
</tr>
</tbody>
</table>

Source: Ford Motor Company

38. Kraft Foods North America
Volume Food Produced in a year

<table>
<thead>
<tr>
<th>Food Group</th>
<th>Volume (in pounds)</th>
<th>Percent (Round to Nearest Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheese, Meals, and Enhancers</td>
<td>6183</td>
<td></td>
</tr>
<tr>
<td>Biscuits, Snacks, and Confectionaries</td>
<td>2083</td>
<td>Example: ( \frac{2083}{13,741} \approx 15% )</td>
</tr>
<tr>
<td>Beverages, Desserts, and Cereals</td>
<td>3905</td>
<td></td>
</tr>
<tr>
<td>Oscar Mayer and Pizza</td>
<td>1570</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>13,741</td>
<td></td>
</tr>
</tbody>
</table>

Source: Kraft Foods, North America

39. Nordstrom advertised a 25%-off sale. If a London Fog coat originally sold for $256, find the decrease in price and the sale price.

40. A gasoline station decreased the price of a $0.95 cola by 15%. Find the decrease in price and the new price.

41. Iceberg lettuce is grown and shipped to stores for about 40 cents a head, and consumers purchase it for about 86 cents a head. Find the percent increase. (Source: Statistical Abstract of the United States)
42. The lettuce consumption per capita in 1980 was about 25.6 pounds, and in 2005 the consumption dropped to about 22.4 pounds. Find the percent decrease. *(Source: Statistical Abstract of the United States)*

43. Smart Cards (cards with an embedded computer chip) have been growing in popularity in recent years. In 2006, about 1900 million Smart Cards were expected to be issued. This represents a 726% increase from the number of cards that were issued in 2001. How many Smart Cards were issued in 2001? Round to the nearest million. *(Source: The Freedonia Group)*

44. Fuel ethanol production is projected to be 10,800 million gallons in 2009. This represents a 44% increase from the number of gallons produced in 2007. How many millions of gallons were produced in 2007? *(Source: Renewable Fuels Association)*

45. How much of an alloy that is 20% copper should be mixed with 200 ounces of an alloy that is 50% copper in order to get an alloy that is 30% copper?

46. How much water should be added to 30 gallons of a solution that is 70% antifreeze in order to get a mixture that is 60% antifreeze?

47. A junior one-day admission to Hershey Park amusement park in Hershey, Pennsylvania, is $27. This price is increased by 70% for (nonsenior) adults. Find the mark-up and the adult price. *(Note: Prices given are approximations.)*

48. The price of a biology book recently increased by 10%. If this book originally cost $99.90, find the mark-up and the new price.

49. By doubling each dimension, the area of a parallelogram increased from 36 square centimeters to 144 square centimeters. Find the percent increase in area.

50. By doubling each dimension, the area of a triangle increased from 6 square miles to 24 square miles. Find the percent increase in area.

51. A company recently downsized its number of employees by 35%. If there are still 78 employees, how many employees were there prior to the layoffs?

52. The average number of children born to each U.S. woman has decreased by 44% since 1920. If this average is now 1.9, find the average in 1920. Round to the nearest tenth.

53. The owner of a local chocolate shop wants to develop a new trail mix. How many pounds of chocolate-covered peanuts worth $5 a pound should be mixed with 10 pounds of granola bites worth $2 a pound to get a mixture worth $3 per pound?

54. A new self-tanning lotion for everyday use is to be sold. First, an experimental lotion mixture is made by mixing 800 ounces of everyday moisturizing lotion worth $0.30 an ounce with self-tanning lotion worth $3 per ounce. If the experimental lotion is to cost $1.20 per ounce, how many ounces of the self-tanning lotion should be in the mixture?

55. The number of farms in the United States was 2.19 million in 2000. By 2006, the number had dropped to 2.09 million. What was the percent of decrease? Round to the nearest tenth of a percent. *(Source: USDA: National Agricultural Statistical Service)*
56. The average size of farms in the United States was 436 acres in 2000. By 2005, the average size had increased to 444 acres. What was the percent increase? Round to the nearest tenth of a percent. \((\text{Source: USDA: National Agricultural Statistical Service})\)

57. The number of Supreme Court decisions has been decreasing in recent years. During the 2005-2006 term, 182 decisions were announced. This is a 45.7% decrease from the number of decisions announced during the 1982-1983 term. How many decisions were announced during 1982-1983? Round to the nearest whole. \((\text{Source: World Almanac})\)

58. The total number of movie screens in the United States has been increasing in recent years. In 2005, there were 37,092 indoor movie screens. This is a 4.3% increase from the number of indoor movie screens in 2000. How many movie screens were operating in 2000? Round to the nearest whole. \((\text{Source: National Association of Theater Owners})\)

59. Scoville units are used to measure the hotness of a pepper. Measuring 577 thousand Scoville units, the “Red Savina” habañero pepper was known as the hottest chili pepper. That has recently changed with the discovery of Naga Jolokia pepper from India. It measures 48% hotter than the habañero. Find the measure of the Naga Jolokia pepper. Round to the nearest thousand units.

60. At this writing, the women’s world record for throwing a disc (like a heavy Frisbee) was set by Jennifer Griffin of the United States in 2000. Her throw was 138.56 meters. His throw was 80.4% farther than Jennifer’s. Find the distance of his throw. Round to the nearest meter. \((\text{Source: World Flying Disc Federation})\)

61. A recent survey showed that 42% of recent college graduates named flexible hours as their most desired employment benefit. In a graduating class of 860 college students, how many would you expect to rank flexible hours as their top priority in job benefits? \((\text{Round to the nearest whole.})\) \((\text{Source: JobTrak.com})\)

62. A recent survey showed that 64% of U.S. colleges have Internet access in their classrooms. There are approximately 9800 post-secondary institutions in the United States. How many of these would you expect to have Internet access in their classrooms? \((\text{Source: Market Data Retrieval, National Center for Education Statistics})\)

\section*{Review and Preview}

\textbf{Place} <, >, or = \textbf{in the appropriate space to make each a true statement. See Sections 1.2, 1.4, and 1.7.}

\begin{align*}
63. & \quad -5 \quad -7 \quad 64. \quad \frac{12}{3} \quad 2^2 \\
65. & \quad -5 \quad -(\quad -5) \quad 66. \quad -3^3 \quad -(\quad -3)^3 \\
67. & \quad (-3)^2 \quad -3^2 \quad 68. \quad -2 \quad -[-2] \quad \hline
\end{align*}

\section*{Concept Extensions}

\begin{align*}
69. \quad \text{Is it possible to mix a 10\% acid solution and a 40\% acid solution to obtain a 60\% acid solution? Why or why not?}
\end{align*}

70. Must the percents in a circle graph have a sum of 100\%? Why or why not?

71. A trail mix is made by combining peanuts worth $3 a pound, raisins worth $2 a pound, and M & M’s worth $4 a pound. Would it make good business sense to sell the trail mix for $1.98 a pound? Why or why not?

72. \begin{align*}
a. \quad \text{Can an item be marked-up by more than 100\%? Why or why not?} \\
b. \quad \text{Can an item be discounted by more than 100\%? Why or why not?}
\end{align*}

Standardized nutrition labels like the one below have been displayed on food items since 1994. The percent column on the right shows the percent of daily values (based on a 2000-calorie diet) shown at the bottom of the label. For example, a serving of this food contains 4 grams of total fat, where the recommended daily fat based on a 2000-calorie diet is less than 65 grams of fat. This means that of or approximately 6\% (as shown) of your daily recommended fat is taken in by eating a serving of this food. Use this nutrition label to answer Exercises 73 through 75.

\begin{table}[h]
\centering
\begin{tabular}{lrr}
\hline
\textbf{Nutrition Facts} & \\
\hline
\multicolumn{2}{c}{\textbf{Serving Size}} & \textbf{18 Crackers (21g)} \\
\multicolumn{2}{c}{\textbf{Servings Per Container}} & \textbf{About 3} \\
\hline
\textbf{Amount Per Serving} & \\
\hline
\textbf{Calories} & 130 & Calories from Fat 35 \\
\textbf{Total Fat} & 4g & \% Daily Value 6\% \\
\textbf{Saturated Fat} & 0.5g & \% Daily Value 3\% \\
\textbf{Polyunsaturated Fat} & 0g & \% Daily Value 0\% \\
\textbf{Total Carbohydrate} & 23g & \% Daily Value 8\% \\
\textbf{Dietary Fiber} & 2g & \% Daily Value 8\% \\
\textbf{Sugars} & 3g & \% Daily Value 8\% \\
\textbf{Protein} & 2g & \% Daily Value 4\% \\
\hline
\textbf{Vitamin A} & - & Vitamin C 0\% \\
\textbf{Calcium} & 2\% & - \\
\textbf{Iron} & 6\% & - \\
\hline
\multicolumn{3}{c}{Different daily values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.} \\
\hline
\textbf{Calories} & 2,000 & 2,500 \\
\hline
\textbf{Total Fat} & Less than 65g & Less than 75g \\
\textbf{Saturated Fat} & Less than 20g & Less than 25g \\
\textbf{Trans Fat} & Less than 2g & Less than 2.5g \\
\textbf{Cholesterol} & Less than 300mg & Less than 350mg \\
\textbf{Sodium} & Less than 2,400mg & Less than 2,400mg \\
\textbf{Total Carbohydrate} & 375g & Less than 375g \\
\textbf{Dietary Fiber} & 25g & Less than 25g \\
\hline
\end{tabular}
\end{table}

73. Based on a 2000-calorie diet, what percent of daily value of sodium is contained in a serving of this food? In other words, find \(x\) in the label. (Round to the nearest tenth of a percent.)

74. Based on a 2000-calorie diet, what percent of daily value of total carbohydrate is contained in a serving of this food? In other words, find \(y\) in the label. (Round to the nearest tenth of a percent.)

75. Notice on the nutrition label that one serving of this food contains 130 calories and 35 of these calories are from fat. Find the percent of calories from fat. (Round to the nearest tenth of a percent.) It is recommended that no more than 30\% of calorie intake come from fat. Does this food satisfy this recommendation?
Use the nutrition label below to answer Exercises 76 through 78.

<table>
<thead>
<tr>
<th>NUTRITIONAL INFORMATION PER SERVING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serving Size: 9.8 oz.</td>
</tr>
<tr>
<td>Servings Per Container: 1</td>
</tr>
<tr>
<td>Calories .................. 280</td>
</tr>
<tr>
<td>Polyunsaturated Fat ........ 1g</td>
</tr>
<tr>
<td>Protein .................. 12g</td>
</tr>
<tr>
<td>Saturated Fat .................. 3g</td>
</tr>
<tr>
<td>Carbohydrate ................. 45g</td>
</tr>
<tr>
<td>Cholesterol ................. 20mg</td>
</tr>
<tr>
<td>Fat .................. 6g</td>
</tr>
<tr>
<td>Sodium .................. 520mg</td>
</tr>
<tr>
<td>Percent of Calories from Fat ....?</td>
</tr>
<tr>
<td>Potassium ................. 220mg</td>
</tr>
</tbody>
</table>

76. If fat contains approximately 9 calories per gram, find the percent of calories from fat in one serving of this food. (Round to the nearest tenth of a percent.)

77. If protein contains approximately 4 calories per gram, find the percent of calories from protein from one serving of this food. (Round to the nearest tenth of a percent.)

78. Find a food that contains more than 30% of its calories per serving from fat. Analyze the nutrition label and verify that the percents shown are correct.

---

**OBJECTIVES**

1. Solve problems involving distance.
2. Solve problems involving money.
3. Solve problems involving interest.

---

**2.7 FURTHER PROBLEM SOLVING**

This section is devoted to solving problems in the categories listed. The same problem-solving steps used in previous sections are also followed in this section. They are listed below for review.

**General Strategy for Problem Solving**

1. **UNDERSTAND** the problem. During this step, become comfortable with the problem. Some ways of doing this are:
   - Read and reread the problem.
   - Choose a variable to represent the unknown.
   - Construct a drawing, whenever possible.
   - Propose a solution and check. Pay careful attention to how you check your proposed solution. This will help writing an equation to model the problem.
2. **TRANSLATE** the problem into an equation.
3. **SOLVE** the equation.
4. **INTERPRET** the results: Check the proposed solution in the stated problem and state your conclusion.

**OBJECTIVE 1** Solving distance problems. Our first example involves distance. For a review of the distance formula, \( d = r \cdot t \), see Section 2.5, Example 1 and the table before the example.

**EXAMPLE 1** Finding Time Given Rate and Distance

Marie Antonio, a bicycling enthusiast, rode her 21-speed at an average speed of 18 miles per hour on level roads and then slowed down to an average of 10 mph on the hilly roads of the trip. If she covered a distance of 98 miles, how long did the entire trip take if traveling the level roads took the same time as traveling the hilly roads?

**Solution**

1. **UNDERSTAND** the problem. To do so, read and reread the problem. The formula \( d = r \cdot t \) is needed. At this time, let’s guess a solution. Suppose that she spent 2 hours traveling on the level roads. This means that she also spent 2 hours traveling on the hilly roads, since the times spent were the same. What is her total distance? Her distance on the level road is rate \( \times \) time = 18(2) = 36 miles. Her distance on the hilly roads is rate \( \times \) time = 10(2) = 20 miles. This gives a total distance of 36 miles + 20 miles = 56 miles, not the correct distance of 98 miles. Remember that the purpose of guessing a solution is not to guess correctly (although this may
happen) but to help better understand the problem and how to model it with an equation. We are looking for the length of the entire trip, so we begin by letting

\[ x = \text{the time spent on level roads.} \]

Because the same amount of time is spent on hilly roads, then also

\[ x = \text{the time spent on hilly roads.} \]

2. **TRANSLATE.** To help us translate to an equation, we now summarize the information from the problem on the following chart. Fill in the rates given, the variables used to represent the times, and use the formula \( d = r \cdot t \) to fill in the distance column.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>18</td>
<td>( x )</td>
</tr>
<tr>
<td>Hilly</td>
<td>10</td>
<td>( x )</td>
</tr>
</tbody>
</table>

Since the entire trip covered 98 miles, we have that

In words: total distance = level distance + hilly distance

Translate: \( 98 = 18x + 10x \)

3. **SOLVE.**

\[
98 = 28x \quad \text{Add like terms.}
\]

\[
98 = 28 \frac{x}{28} = \frac{28x}{28} \quad \text{Divide both sides by 28.}
\]

\[
x = 3.5
\]

4. **INTERPRET the results.**

**Check:** Recall that \( x \) represents the time spent on the level portion of the trip and also the time spent on the hilly portion. If Marie rides for 3.5 hours at 18 mph, her distance is \( 18(3.5) = 63 \) miles. If Marie rides for 3.5 hours at 10 mph, her distance is \( 10(3.5) = 35 \) miles. The total distance is 63 miles + 35 miles = 98 miles, the required distance.

**State:** The time of the entire trip is then 3.5 hours + 3.5 hours or 7 hours.

**PRACTICE**

Sat Tranh took a short hike with his friends up Mt. Wachusett. They hiked uphill at a steady pace of 1.5 miles per hour, and downhill at a rate of 4 miles per hour. If the time to climb the mountain took an hour more than the time to hike down, how long was the entire hike?

**EXAMPLE 2** **Finding Train Speeds**

The Kansas City Southern Railway operates in 10 states and Mexico. Suppose two trains leave Neosho, Missouri, at the same time. One travels north and the other travels south at a speed that is 15 miles per hour faster. In 2 hours, the trains are 230 miles apart. Find the speed of each train.

---

KS

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Distribution
CHAPTER 2 Equations, Inequalities, and Problem Solving

Solution

1. UNDERSTAND the problem. Read and reread the problem. Guess a solution and check. Let’s let

\[ x = \text{speed of train traveling north} \]

Because the train traveling south is 15 mph faster, we have

\[ x + 15 = \text{speed of train traveling south} \]

2. TRANSLATE. Just as for Example 1, let’s summarize our information on a chart. Use the formula \( d = r \cdot t \) to fill in the distance column.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( t )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Train</td>
<td>( x )</td>
<td>2</td>
<td>( 2x )</td>
</tr>
<tr>
<td>South Train</td>
<td>( x + 15 )</td>
<td>2</td>
<td>( 2(x + 15) )</td>
</tr>
</tbody>
</table>

Since the total distance between the trains is 230 miles, we have

\[
2x + 2(x + 15) = 230
\]

3. SOLVE

\[
2x + 2x + 30 = 230
\]

Use the distributive property.

\[
4x + 30 = 230
\]

Combine like terms.

\[
4x = 200
\]

Subtract 30 from both sides.

\[
\frac{4x}{4} = \frac{200}{4}
\]

Divide both sides by 4.

\[
x = 50
\]

Simplify.

4. INTERPRET the results.

Check: Recall that \( x \) is the speed of the train traveling north, or 50 mph. In 2 hours, this train travels a distance of \( 2(50) = 100 \) miles. The speed of the train traveling south is \( x + 15 \) or \( 50 + 15 = 65 \) mph. In 2 hours, this train travels \( 2(65) = 130 \) miles. The total distance of the trains is 100 miles + 130 miles = 230 miles, the required distance.

State: The northbound train’s speed is 50 mph and the southbound train’s speed is 65 mph.

OBJECTIVE 2 Solving money problems. The next example has to do with finding an unknown number of a certain denomination of coin or bill. These problems are extremely useful in that they help you understand the difference between the number of coins or bills and the total value of the money.

For example, suppose there are seven $5-bills. The number of $5-bills is 7 and the total value of the money is $5(7) = $35.
Study the table below for more examples.

<table>
<thead>
<tr>
<th>Denomination of Coin or Bill</th>
<th>Number of Coins or Bills</th>
<th>Value of Coins or Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-dollar bills</td>
<td>17</td>
<td>$20(17) = $340</td>
</tr>
<tr>
<td>nickels</td>
<td>31</td>
<td>$0.05(31) = $1.55</td>
</tr>
<tr>
<td>quarters</td>
<td>$x$</td>
<td>$0.25(x) = 0.25x$</td>
</tr>
</tbody>
</table>

**EXAMPLE 3**  
Finding Numbers of Denominations

Part of the proceeds from a local talent show was $2420 worth of $10 and $20 bills. If there were 37 more $20 bills than $10 bills, find the number of each denomination.

**Solution**

1. **UNDERSTAND** the problem. To do so, read and reread the problem. If you’d like, let’s guess a solution. Suppose that there are 25 $10 bills. Since there are 37 more $20 bills, we have 25 + 37 = 62 $20 bills. The total amount of money is $10(25) + $20(62) = $1490, below the given amount of $2420. Remember that our purpose for guessing is to help us better understand the problem.

   We are looking for the number of each denomination, so we let

   \[ x = \text{number of } $10 \text{ bills} \]

   There are 37 more $20 bills, so

   \[ x + 37 = \text{number of } $20 \text{ bills} \]

2. **TRANSLATE**. To help us translate to an equation, study the table below

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Number of Bills</th>
<th>Value of Bills (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 bills</td>
<td>$x$</td>
<td>10$x$</td>
</tr>
<tr>
<td>$20 bills</td>
<td>$x + 37$</td>
<td>20(x + 37)</td>
</tr>
</tbody>
</table>

Since the total value of these bills is $2420, we have

In words:  
value of $10 bills plus value of $20 bills is 2420

Translate:  
10$x$ + 20(x + 37) = 2420

3. **SOLVE**:  

\[
10x + 20x + 740 = 2420 \\
30x + 740 = 2420 \\
30x = 1680 \\
\frac{30x}{30} = \frac{1680}{30} \\
x = 56
\]

Use the distributive property.  
Add like terms.  
Subtract 740 from both sides.  
Divide both sides by 30.

4. **INTERPRET** the results.

**Check**: Since \(x\) represents the number of $10 bills, we have 56 $10 bills and 56 + 37, or 93 $20 bills. The total amount of these bills is $10(56) + $20(93) = $2420, the correct total.

**State**: There are 56 $10 bills and 93 $20 bills.
A stack of $5 and $20 bills was counted by the treasurer of an organization. The total value of the money was $1710 and there were 47 more $5 bills than $20 bills. Find the number of each type of bill.

OBJECTIVE 3  Solving interest problems. The next example is an investment problem. For a review of the simple interest formula, \( I = PRT \), see the table at the beginning of Section 2.5 and also Exercises 11 and 12 in that exercise set.

**EXAMPLE 4**  Finding the Investment Amount

Rajiv Puri invested part of his $20,000 inheritance in a mutual funds account that pays 7% simple interest yearly and the rest in a certificate of deposit that pays 9% simple interest yearly. At the end of one year, Rajiv’s investments earned $1550. Find the amount he invested at each rate.

**Solution**

1. **UNDERSTAND.** Read and reread the problem. Next, guess a solution. Suppose that Rajiv invested $8000 in the 7% fund and the rest, $12,000, in the fund paying 9%. To check, find his interest after one year. Recall the formula, \( I = PRT \), so the interest from the 7% fund = \( 8000(0.07)(1) = 560 \). The interest from the 9% fund = \( 12,000(0.09)(1) = 1080 \). The sum of the interests is \( 560 + 1080 = 1640 \). Our guess is incorrect, since the sum of the interests is not $1550, but we now have a better understanding of the problem.

Let

- \( x \) = amount of money in the account paying 7%.
- The rest of the money is $20,000 less \( x \) or \( 20,000 - x \) = amount of money in the account paying 9%.

2. **TRANSLATE.** We apply the simple interest formula \( I = PRT \) and organize our information in the following chart. Since there are two different rates of interest and two different amounts invested, we apply the formula twice.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Time</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7% Fund</strong></td>
<td>( x )</td>
<td>0.07</td>
<td>1</td>
</tr>
<tr>
<td><strong>9% Fund</strong></td>
<td>( 20,000 - x )</td>
<td>0.09</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>20,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The total interest earned, $1550, is the sum of the interest earned at 7% and the interest earned at 9%.

In words: \( \text{interest at 7%} + \text{interest at 9%} = \text{total interest} \)

Translate: \( 0.07x + 0.09(20,000 - x) = 1550 \)

3. **SOLVE.**

\[
0.07x + 0.09(20,000 - x) = 1550 \\
0.07x + 1800 - 0.09x = 1550 \quad \text{Apply the distributive property.} \\
1800 - 0.02x = 1550 \quad \text{Combine like terms.} \\
-0.02x = -250 \quad \text{Subtract 1800 from both sides.} \\
x = 12,500 \quad \text{Divide both sides by } -0.02.
\]
Section 2.7 Further Problem Solving

4. INTERPRET.
Check: If $x = 12,500$, then $20,000 - x = 20,000 - 12,500$ or 7500. These solutions are reasonable, since their sum is $20,000$ as required. The annual interest on $12,500 at 7% is $875; the annual interest on $7500 at 9% is $675, and $875 + $675 = $1550.

State: The amount invested at 7% is $12,500. The amount invested at 9% is $7500.

Solve. See Examples 1 and 2.

1. A jet plane traveling at 500 mph overtakes a propeller plane traveling at 200 mph that had a 2-hour head start. How far from the starting point are the planes?

2. How long will it take a bus traveling at 60 miles per hour to overtake a car traveling at 40 mph if the car had a 1.5-hour head start?

3. A bus traveled on a level road for 3 hours at an average speed 20 miles per hour faster than it traveled on a winding road. The time spent on the winding road was 4 hours. Find the average speed on the level road if the entire trip was 305 miles.

4. The Jones family drove to Disneyland at 50 miles per hour and returned on the same route at 40 mph. Find the distance to Disneyland if the total driving time was 7.2 hours.

Complete the table. The first and sixth rows have been completed for you. See Example 3.

<table>
<thead>
<tr>
<th>Number of Bills</th>
<th>Value of Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 bills</td>
<td></td>
</tr>
<tr>
<td>$10 bills</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

13. Part of the proceeds from a garage sale was $280 worth of $5 and $10 bills. If there were 20 more $5 bills than $10 bills, find the number of each denomination.

<table>
<thead>
<tr>
<th>Number of Bills</th>
<th>Value of Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20 bills</td>
<td></td>
</tr>
<tr>
<td>$50 bills</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

14. A bank teller is counting $20 and $50-dollar bills. If there are six times as many $20 bills as $50 bills and the total amount of money is $3910, find the number of each denomination.

<table>
<thead>
<tr>
<th>Number of Bills</th>
<th>Value of Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20 bills</td>
<td></td>
</tr>
<tr>
<td>$50 bills</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

Solve. See Example 4.

15. Zoya Lon invested part of her $25,000 advance at 8% annual simple interest and the rest at 9% annual simple interest. If her total yearly interest from both accounts was $2135, find the amount invested at each rate.

16. Karen Waughtal invested some money at 9% annual simple interest and $250 more than that amount at 10% annual simple interest. If her total yearly interest was $101, how much was invested at each rate?

17. Sam Mathius invested part of his $10,000 bonus in a fund that paid an 11% profit and invested the rest in stock that suffered a 4% loss. Find the amount of each investment if his overall net profit was $650.

18. Bruce Blossum invested a sum of money at 10% annual simple interest and invested twice that amount at 12% annual interest.
simple interest. If his total yearly income from both investments was $2890, how much was invested at each rate?

19. The Concordia Theatre contains 500 seats and the ticket prices for a recent play were $43 for adults and $28 for children. For one matinee, if the total proceeds were $16,805, how many of each type of ticket were sold?

20. A zoo in Oklahoma charged $22 for adults and $15 for children. During a summer day, 732 zoo tickets were sold and the total receipts were $12,912. How many children and how many adults tickets were sold?

MIXED PRACTICE

21. How can $54,000 be invested, part at 8% annual simple interest and the remainder at 10% annual simple interest, so that the interest earned by the two accounts will be equal?

22. Ms. Mills invested her $20,000 bonus in two accounts. She took a 4% loss on one investment and made a 12% profit on another investment, but ended up breaking even. How much was invested in each account?

23. Alan and Dave Schaferkötter leave from the same point hiking in opposite directions, Alan driving at 55 miles per hour and Dave at 65 mph. Alan has a one-hour head start. How long will they be able to talk on their car phones if the phones have a 250-mile range?

24. Kathleen and Cade Williams leave simultaneously from the same point hiking in opposite directions, Kathleen walking at 4 miles per hour and Cade at 5 mph. How long can they talk on their walkie-talkies if the walkie-talkies have a 20-mile radius?

25. A youth organization collected nickels and dimes for a charity drive. By the end of the 1-day drive, the youth had collected $56.35. If there were three times as many dimes as nickels, how many of each type of coin was collected?

26. A collection of dimes and quarters are retrieved from a soft drink machine. There are five times as many dimes as quarters and the total value of the coins is $27.75. Find the number of dimes and the number of quarters.

27. If $3000 is invested at 6% annual simple interest, how much should be invested at 9% annual simple interest so that the total yearly income from both investments is $585?

28. Trudy Waterbury, a financial planner, invested a certain amount of money at 9% annual simple interest, twice that amount at 10% annual simple interest, and three times that amount at 11% annual simple interest. Find the amount invested at each rate if her total yearly income from the investments was $2790.

29. Two hikers are 11 miles apart and walking toward each other. They meet in 2 hours. Find the rate of each hiker if one hiker walks 1.1 mph faster than the other.

30. Nedra and Latonya Dominguez are 12 miles apart hiking toward each other. How long will it take them to meet if Nedra walks at 3 mph and Latonya walks 1 mph faster?

31. Mark Martin can row upstream at 5 mph and downstream at 11 mph. If Mark starts rowing upstream until he gets tired and then rows downstream to his starting point, how far did Mark row if the entire trip took 4 hours?

32. On a 255-mile trip, Gary Alessandrini traveled at an average speed of 70 mph, got a speeding ticket, and then traveled at 60 mph for the remainder of the trip. If the entire trip took 4.5 hours and the speeding ticket stop took 30 minutes, how long did Gary speed before getting stopped?

REVIEW AND PREVIEW

Perform the indicated operations. See Sections 1.5 and 1.6.

33. $3 + (-7) \quad \quad \quad \quad \quad \quad 34. (-2) + (-8)$

35. $\frac{3}{4} - \frac{3}{16}$

37. $-5 - (-1)$

38. $-12 - 3$

CONCEPT EXTENSIONS

39. A stack of $20, $50, and $100 bills was retrieved as part of an FBI investigation. There were 46 more $50 bills than $100 bills. Also, the number of $20 bills was 7 times the number of $100 bills. If the total value of the money was $9550, find the number of each type of bill.

40. A man places his pocket change in a jar every day. The jar is full and his children have counted the change. The total value is $44.86. Let $x$ represent the number of quarters and use the information below to find the number of each type of coin.

There are: 136 more dimes than quarters
8 times as many nickels as quarters
32 more than 16 times as many pennies as quarters

To "break even" in a manufacturing business, revenue $R$ (income) must equal the cost $C$ of production, or $R = C$.

41. The cost $C$ to produce $x$ number of skateboards is given by $C = 100 + 20x$. The skateboards are sold wholesale for $24 each, so revenue $R$ is given by $R = 24x$. Find how many skateboards the manufacturer needs to produce and sell to break even. (Hint: Set the expression for $R$ equal to the expression for $C$, then solve for $x$.)

42. The revenue $R$ from selling $x$ number of computer boards is given by $R = 60x$, and the cost $C$ of producing them is given by $C = 50x + 5000$. Find how many boards must be sold to break even. Find how much money is needed to produce the break-even number of boards.

43. The cost $C$ of producing $x$ number of paperback books is given by $C = 4.50x + 2400$. Income $R$ from these books is given by $R = 7.50x$. Find how many books should be produced and sold to break even.

44. Find the break-even quantity for a company that makes $x$ number of computer monitors at a cost $C$ given by $C = 870 + 70x$ and receives revenue $R$ given by $R = 105x$.

45. Exercises 41 through 44 involve finding the break-even point for manufacturing. Discuss what happens if a company makes and sells fewer products than the break-even point. Discuss what happens if more products than the break-even point are made and sold.
This definition and all other definitions, properties, and steps in this section also hold true for the inequality symbols, $>$, $\geq$, and $\leq$.

A **solution of an inequality** is a value of the variable that makes the inequality a true statement. The solution set is the set of all solutions. For the inequality $x < 3$, replacing $x$ with any number less than 3, that is, to the left of 3 on a number line, makes the resulting inequality true. This means that any number less than 3 is a solution of the inequality $x < 3$. 

**Linear Inequality in One Variable**

A **linear inequality in one variable** is an inequality that can be written in the form $ax + b < c$

where $a$, $b$, and $c$ are real numbers and $a$ is not 0.

**Answer the following questions based on your most recent mathematics exam, whenever that was.**

1. How soon before class did you arrive?
2. Did you read the directions on the test carefully?
3. Did you make sure you answered the question asked for each problem on the exam?
4. Were you able to attempt each problem on your exam?
5. If your answer to question 4 is no, list reasons why.
6. Did you have extra time on your exam?
7. If your answer to question 6 is yes, describe how you spent that extra time.

**Good luck!**
Since there are infinitely many such numbers, we cannot list all the solutions of the inequality. We can use set notation and write

\[{ x \mid x < 3}\]. Recall that this is read

the set of all \(x\) such that \(x\) is less than 3.

We can also picture the solutions on a number line. If we use open/closed-circle notation, the graph of \(x < 3\) looks like the following.

In this text, a convenient notation, called "interval notation," will be used to write solution sets of inequalities. To help us understand this notation, a different graphing notation will be used. Instead of an open circle, we use a parenthesis; instead of a closed circle, we use a bracket. With this new notation, the graph of \(x < 3\) now looks like and can be represented in interval notation as \((-\infty, 3]\). The symbol \(-\infty\), read as "negative infinity," does not indicate a number, but does indicate that the shaded arrow to the left never ends. In other words, the interval \((-\infty, 3]\) includes all numbers less than 3.

Picturing the solutions of an inequality on a number line is called graphing the solutions or graphing the inequality, and the picture is called the graph of the inequality.

To graph \(x \leq 3\) or simply \(x \leq 3\), shade the numbers to the left of 3 and place a bracket at 3 on the number line. The bracket indicates that 3 is a solution: 3 is less than or equal to 3. In interval notation, we write \((-\infty, 3]\).

**Helpful Hint**
When writing an inequality in interval notation, it may be easier to first graph the inequality, then write it in interval notation. To help think of the number line as approaching \(-\infty\) to the left and \(+\infty\) or \(\infty\) to the right. Then simply write the interval notation by following your shading from left to right.

**EXAMPLE 1** Graph \(x \geq -1\). Then write the solutions in interval notation.

**Solution** We place a bracket at \(-1\) since the inequality symbol is \(\geq\) and \(-1\) is greater than or equal to \(-1\). Then we shade to the right of \(-1\).

In interval notation, this is \([-1, \infty)\).

**PRACTICE 1** Graph \(x < 5\). Then write the solutions in interval notation.
OBJECTIVE 2 ► Solving linear inequalities. When solutions of a linear inequality are not immediately obvious, they are found through a process similar to the one used to solve a linear equation. Our goal is to get the variable alone, and we use properties of inequality similar to properties of equality.

**Addition Property of Inequality**

If $a$, $b$, and $c$ are real numbers, then

$$a < b \quad \text{and} \quad a + c < b + c$$

are equivalent inequalities.

This property also holds true for subtracting values, since subtraction is defined in terms of addition. In other words, adding or subtracting the same quantity from both sides of an inequality does not change the solution of the inequality.

**EXAMPLE 2** Solve $x + 4 \leq -6$ for $x$. Graph the solution set and write it in interval notation.

**Solution**

To solve for $x$, subtract 4 from both sides of the inequality.

1. Original inequality:
   $$x + 4 \leq -6$$
2. Subtract 4 from both sides:
   $$x + 4 - 4 \leq -6 - 4$$
3. Simplify:
   $$x \leq -10$$

The solution set is $(-\infty, -10]$.

**Practice 2** Solve $x + 11 \geq 6$. Graph the solution set and write it in interval notation.

**Helpful Hint**

Notice that any number less than or equal to $-10$ is a solution to $x \leq -10$. For example, solutions include

$$-10, -200, -11\frac{1}{2}, -\pi, -\sqrt{130}, -50.3$$

An important difference between linear equations and linear inequalities is shown when we multiply or divide both sides of an inequality by a nonzero real number. For example, start with the true statement $6 < 8$ and multiply both sides by 2. As we see below, the resulting inequality is also true.

$$6 < 8 \quad \text{True}$$
$$2(6) < 2(8) \quad \text{Multiply both sides by 2.}$$
$$12 < 16 \quad \text{True}$$

But if we start with the same true statement $6 < 8$ and multiply both sides by $-2$, the resulting inequality is not a true statement.

$$6 < 8 \quad \text{True}$$
$$-2(6) < -2(8) \quad \text{Multiply both sides by } -2.$$  
$$-12 < -16 \quad \text{False}$$

Notice, however, that if we reverse the direction of the inequality symbol, the resulting inequality is true.

$$-12 < -16 \quad \text{False}$$
$$-12 > -16 \quad \text{True}$$
This demonstrates the multiplication property of inequality.

**Multiplication Property of Inequality**

1. If \(a, b,\) and \(c\) are real numbers, and \(c\) is **positive**, then
   \[
   a < b \quad \text{and} \quad ac < bc
   \]
   are equivalent inequalities.

2. If \(a, b,\) and \(c\) are real numbers, and \(c\) is **negative**, then
   \[
   a < b \quad \text{and} \quad ac > bc
   \]
   are equivalent inequalities.

Because division is defined in terms of multiplication, this property also holds true when dividing both sides of an inequality by a nonzero number. If we multiply or divide both sides of an inequality by a negative number, the direction of the inequality sign **must be reversed** for the inequalities to remain equivalent.

**Example 3** Solve \(-2x \leq -4\). Graph the solution set and write it in interval notation.

**Solution** Remember to reverse the direction of the inequality symbol when dividing by a negative number.

\[
\begin{align*}
-2x & \leq -4 \\
\frac{-2x}{-2} & \geq \frac{-4}{-2} \\
x & \geq 2
\end{align*}
\]

Simplify.

The solution set \([2, \infty)\) is graphed as shown.

**Practice 3** Solve \(-5x \geq -15\). Graph the solution set and write it in interval notation.

**Example 4** Solve \(2x < -4\). Graph the solution set and write it in interval notation.

**Solution**

\[
\begin{align*}
2x & < -4 \\
\frac{2x}{2} & < \frac{-4}{2} \\
x & < -2
\end{align*}
\]

Do not reverse the direction of the inequality sign.

The solution set \((-\infty, -2)\) is graphed as shown.
Section 2.8 Solving Linear Inequalities

4. Solve $3x > -9$. Graph the solution set and write it in interval notation.

Concept Check ✓ Fill in the blank with $<, >, \leq$, or $\geq$.

- a. Since $-8 < -4$, then $3(-8) \underline{\hphantom{<}} 3(-4)$.
- b. Since $5 \geq -2$, then $\frac{5}{7} \underline{\hphantom{<}} \frac{-2}{7}$.
- c. If $a < b$, then $2a \underline{\hphantom{<}} 2b$.
- d. If $a \geq b$, then $\frac{a}{-3} \underline{\hphantom{<}} \frac{b}{-3}$.

The following steps may be helpful when solving inequalities. Notice that these steps are similar to the ones given in Section 2.3 for solving equations.

Solving Linear Inequalities in One Variable

1. Clear the inequality of fractions by multiplying both sides of the inequality by the lowest common denominator (LCD) of all fractions in the inequality.
2. Remove grouping symbols such as parentheses by using the distributive property.
3. Simplify each side of the inequality by combining like terms.
4. Write the inequality with variable terms on one side and numbers on the other side by using the addition property of inequality.
5. Get the variable alone by using the multiplication property of inequality.

 Helpful Hint Don’t forget that if both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality sign must be reversed.

Example 5 Solve $-4x + 7 \geq -9$. Graph the solution set and write it in interval notation.

Solution

\[-4x + 7 \geq -9\]

\[-4x + 7 - 7 \geq -9 - 7\] Subtract 7 from both sides.

\[-4x \geq -16\] Simplify.

\[-4x \underline{\hphantom{<}} -16\] Divide both sides by $-4$ and reverse the direction of the inequality sign.

\[-4 \underline{\hphantom{<}} -4\] Simplify.

\[x \leq 4\]

The solution set $(-\infty, 4]$ is graphed as shown.

Answers to Concept Check:

- a. $<$
- b. $\leq$
- c. $<$
- d. $\leq$
EXAMPLE 6  Solve \(2x + 7 \leq x - 11\). Graph the solution set and write it in interval notation.

Solution
\[
2x + 7 \leq x - 11 \\
2x + 7 - x \leq x - 11 - x \\
x + 7 \leq -11 \\
x + 7 - 7 \leq -11 - 7 \\
x \leq -18
\]

The graph of the solution set \((-\infty, -18]\) is shown.

EXAMPLE 7  Solve \(-5x + 7 < 2(x - 3)\). Graph the solution set and write it in interval notation.

Solution
\[
-5x + 7 < 2(x - 3) \\
-5x + 7 < 2x - 6 \\
-5x + 7 - 2x < 2x - 6 - 2x \\
-7x + 7 < -6 \\
-7x + 7 - 7 < -6 - 7 \\
-7x < -13 \\
\frac{-7x}{-7} > \frac{-13}{-7} \\
x > \frac{13}{7}
\]

The graph of the solution set \(\left(\frac{13}{7}, \infty\right)\) is shown.

EXAMPLE 8  Solve \(2(x - 3) - 5 \leq 3(x + 2) - 18\). Graph the solution set and write it in interval notation.

Solution
\[
2(x - 3) - 5 \leq 3(x + 2) - 18 \\
2x - 6 - 5 \leq 3x + 6 - 18 \\
2x - 11 \leq 3x - 12 \\
-x - 11 \leq -12 \\
x \leq -1
\]
Section 2.8 Solving Linear Inequalities

Divide both sides by \(-1\) and reverse the direction of the inequality sign.

\[
\frac{-x}{-1} \geq \frac{-1}{-1}
\]

Simplify.

\[
x \geq 1
\]

The graph of the solution set \([1, \infty)\) is shown.

**Practice**

Solve \(3(x - 4) - 5 \leq 5(x - 1) - 12\). Graph the solution set and write it in interval notation.

**OBJECTIVE 3** Solving compound inequalities. Inequalities containing one inequality symbol are called simple inequalities, while inequalities containing two inequality symbols are called compound inequalities. A compound inequality is really two simple inequalities in one. The compound inequality

\[
3 < x < 5
\]

means \(3 < x \text{ and } x < 5\).

This can be read “\(x\) is greater than 3 and less than 5.”

A solution of a compound inequality is a value that is a solution of both of the simple inequalities that make up the compound inequality. For example,

\[
4\frac{1}{2}
\]

is a solution of \(3 < x < 5\) since \(3 < 4\frac{1}{2} \text{ and } 4\frac{1}{2} < 5\).

To graph \(3 < x < 5\), place parentheses at both 3 and 5 and shade between.

**Example 9**

Graph \(2 < x \leq 4\). Write the solutions in interval notation.

**Solution**

Graph all numbers greater than 2 and less than or equal to 4. Place a parenthesis at 2, a bracket at 4, and shade between.

In interval notation, this is \((2, 4]\).

**Practice**

Graph \(-3 \leq x < 1\). Write the solutions in interval notation.

When we solve a simple inequality, we isolate the variable on one side of the inequality. When we solve a compound inequality, we isolate the variable in the middle part of the inequality. Also, when solving a compound inequality, we must perform the same operation to all three parts of the inequality: left, middle, and right.

**Example 10**

Solve \(-1 \leq 2x - 3 < 5\). Graph the solution set and write it in interval notation.

**Solution**

\[
-1 \leq 2x - 3 < 5
\]

Add 3 to all three parts.

\[
2 \leq 2x < 8
\]

Combine like terms.

\[
2 \leq \frac{2x}{2} < \frac{8}{2}
\]

Divide all three parts by 2.

\[
1 \leq x < 4
\]

Simplify.
CHAPTER 2 Equations, Inequalities, and Problem Solving

The graph of the solution set \([1, 4)\) is shown.

\[
\begin{array}{c|c|c|c|c|c}
\hline
-2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
\end{array}
\]

**PRACTICE**

**10** Solve \(-4 < 3x + 2 \leq 8\). Graph the solution set and write it in interval notation.

**EXAMPLE 11** Solve \(3 \leq \frac{3x}{2} + 4 \leq 5\). Graph the solution set and write it in interval notation.

**Solution**

\[
3 \leq \frac{3x}{2} + 4 \leq 5
\]

\[
2(3) \leq 2\left(\frac{3x}{2} + 4\right) \leq 2(5)
\]

\[
6 \leq 3x + 8 \leq 10
\]

\[
-2 \leq 3x \leq 2
\]

\[
\frac{-2}{3} \leq \frac{3x}{2} \leq \frac{2}{3}
\]

\[
\frac{2}{3} \leq x \leq \frac{2}{3}
\]

The graph of the solution set \([\frac{-2}{3}, \frac{2}{3}]\) is shown.

**PRACTICE**

**11** Solve \(1 < \frac{3}{4}x + 5 \leq 6\). Graph the solution set and write it in interval notation.

**OBJECTIVE 4** Solving inequality applications. Problems containing words such as “at least,” “at most,” “between,” “no more than,” and “no less than” usually indicate that an inequality should be solved instead of an equation. In solving applications involving linear inequalities, use the same procedure you use to solve applications involving linear equations.

**EXAMPLE 12** Staying within Budget

Marie Chase and Jonathan Edwards are having their wedding reception at the Gallery Reception Hall. They may spend at most \$2000 for the reception. If the reception hall charges a \$100 cleanup fee plus \$36 per person, find the greatest number of people that they can invite and still stay within their budget.

**Solution**

1. UNDERSTAND. Read and reread the problem. Next, guess a solution. If 40 people attend the reception, the cost is \$100 + \$36(40) = \$100 + \$1440 = \$1540. Let \(x\) = the number of people who attend the reception.
Section 2.8 Solving Linear Inequalities

2. TRANSLATE.

In words: cleanup fee + cost per person must be less than or equal to $2000

Translate: 100 + 36x ≤ 2000

3. SOLVE.

100 + 36x ≤ 2000
36x ≤ 1900
x ≤ \frac{7}{9}

4. INTERPRET.

Check: Since x represents the number of people, we round down to the nearest whole, or 52. Notice that if 52 people attend, the cost is $100 + 36(52) = 1972. If 53 people attend, the cost is $100 + 36(53) = 2008, which is more than the given $2000.

State: Marie Chase and Jonathan Edwards can invite at most 52 people to the reception.

Kasonga is eager to begin his education at his local community college. He has budgeted $1500 for college this semester. His local college charges a $300 matriculation fee and costs an average of $375 for tuition, fees, and books for each three-credit course. Find the greatest number of classes Kasonga can afford to take this semester.

PRACTICE

$100 + 36 \cdot 52 = 1972, \quad 100 + 36 \cdot 53 = 2008.$

VOCABULARY & READINESS CHECK

Use the choices below to fill in each blank.

<table>
<thead>
<tr>
<th>expression</th>
<th>inequality</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 6x - 7(x + 9)</td>
<td></td>
<td>2. 6x = 7(x + 9)</td>
</tr>
<tr>
<td>3. 6x &lt; 7(x + 9)</td>
<td></td>
<td>4. 5y - 2 ≥ -38</td>
</tr>
<tr>
<td>5. \frac{9}{7} = \frac{x + 2}{14}</td>
<td></td>
<td>6. \frac{9}{7} - \frac{x + 2}{14}</td>
</tr>
</tbody>
</table>

Decide which number listed is not a solution to each given inequality.

7. x ≥ -3; -3, 0, -5, π  
8. x < 6; -6, -6, 0, -3.2  
9. x < 4.01; 4, -4.01, 4.1, -4.1  
10. x ≥ -3; -4, -3, -2, (-2)  

2.8 EXERCISE SET

Graph each set of numbers given in interval notation. Then write an inequality statement in x describing the numbers graphed.

Graph each inequality on a number line. Then write the solutions in interval notation. See Example 1.

1. [2, ∞)  
2. (-3, ∞)  
3. (-∞, -5)  
4. (-∞, 4]  
5. x ≤ -1  
6. y < 0  
7. x < \frac{1}{2}  
8. z < -\frac{2}{3}
9. $y \geq 5$
10. $x > 3$

Solve each inequality. Graph the solution set and write it in interval notation. See Examples 2 through 4.

11. $2x < -6$
12. $3x > -9$
13. $x - 2 \geq -7$
14. $x + 4 \leq 1$
15. $-8x \leq 16$
16. $-5x < 20$

Solve each inequality. Graph the solution set and write it in interval notation. See Examples 5 and 6.

17. $3x - 5 > 2x - 8$
18. $3 - 7x \geq 10 - 8x$
19. $4x - 1 \leq 5x - 2x$
20. $7x + 3 < 9x - 3x$

Solve each inequality. Graph the solution set and write it in interval notation. See Examples 7 and 8.

21. $x - 7 < 3(x + 1)$
22. $3x + 9 \leq 5(x - 1)$
23. $-6x + 2 \leq 2(5 - x)$
24. $-7x + 4 > 2(4 - x)$
25. $4(3x - 1) \leq 5(2x - 4)$
26. $3(5x - 4) \leq 4(3x - 2)$
27. $3(x + 2) - 6 > -2(x - 3) + 14$
28. $7(x - 2) + x \leq -4(5 - x) - 12$

MIXED PRACTICE

Solve the following inequalities. Graph each solution set and write it in interval notation.

29. $-2x \leq -40$
30. $-7x > 21$
31. $-9 + x > 7$
32. $y - 4 \leq 1$
33. $3x - 7 < 6x + 2$
34. $2x - 1 \geq 4x - 5$
35. $5x - 7x \geq x + 2$
36. $4 - x < 8x + 2x$
37. $\frac{3}{4}x > 2$
38. $\frac{5}{6}x \geq -8$
39. $3(x - 5) < 2(2x - 1)$
40. $5(x + 4) < 4(2x + 3)$
41. $4(2x + 1) < 4$
42. $6(2 - x) \geq 12$
43. $-5x + 4 \geq -4(x - 1)$
44. $-6x + 2 \leq -3(x + 4)$
45. $-2(x - 4) - 3x < -(4x + 1) + 2x$
46. $-5(1 - x) + x = -(6 - 2x) + 6$
47. $-3x + 6 \geq 2x + 6$
48. $-(x - 4) < 4$
49. Explain how solving a linear inequality is similar to solving a linear equation.

50. Explain how solving a linear inequality is different from solving a linear equation.

Graph each inequality. Then write the solutions in interval notation. See Example 9.

51. $-1 < x < 3$
52. $2 \leq y < 3$
53. $0 \leq y < 2$
54. $-1 \leq x < 4$

Solve each inequality. Graph the solution set and write it in interval notation. See Examples 10 and 11.

55. $-3 < 3x < 6$
56. $-5 < 2x < -2$
57. $2 \leq 3x - 10 \leq 5$
58. $4 \leq 5x - 6 \leq 19$
59. $-4 < 2(x - 3) \leq 4$
60. $0 < 4(x + 5) \leq 8$
61. $-2 < 3x - 5 < 7$
62. $1 < 4 + 2x \leq 7$
63. $-6 < 3(x - 2) \leq 8$
64. $-5 \leq 2(x + 4) < 8$
65. Explain how solving a linear inequality is different from solving a compound inequality.

66. Explain how solving a linear inequality is similar to solving a compound inequality.

Solve. See Example 12.

67. Six more than twice a number is greater than negative fourteen. Find all numbers that make this statement true.

68. Five times a number, increased by one, is less than or equal to ten. Find all such numbers.

69. Dennis and Nancy Wood are celebrating their 30th wedding anniversary by having a reception at Tiffany Oaks reception hall. They have budgeted $3000 for their reception. If the reception hall charges a $50.00 cleanup fee plus $34 per person, find the greatest number of people that may be invited and still stay within their budget.

70. A surprise retirement party is being planned for Pratep Puri. A total of $860 has been collected for the event, which is to be held at a local reception hall. This reception hall charges a cleanup fee of $40 and $15 per person for drinks and light snacks. Find the greatest number of people that may be invited and still stay within $860.

71. Find the values for \( x \) so that the perimeter of this rectangle is no greater than 100 centimeters.

72. Find the values for \( x \) so that the perimeter of this triangle is no longer than 87 inches.

73. A financial planner has a client with $15,000 to invest. If he invests $10,000 in a certificate of deposit paying 11% annual simple interest, at what rate does the remainder of the money need to be invested so that the two investments together yield at least $1600 in yearly interest?

74. Alex earns $600 per month plus 4% of all his sales over $1000. Find the minimum sales that will allow Alex to earn at least $3000 per month.

75. Ben Holladay bowled 146 and 201 in his first two games. What must he bowl in his third game to have an average of at least 180?

76. On an NBA team the two forwards measure 6'8" and 6'6" and the two guards measure 6'0" and 5'9" tall. How tall a center should they hire if they wish to have a starting team average height of at least 6'5"?

77. High blood cholesterol levels increase the risk of heart disease in adults. Doctors recommend that total blood cholesterol be less than 200 milligrams per deciliter. Total cholesterol levels from 200 up to 240 milligrams per deciliter are considered borderline. Any total cholesterol reading above 240 milligrams per deciliter is considered high.

Letting \( x \) represent a patient's total blood cholesterol level, write a series of three inequalities that describe the ranges corresponding to recommended, borderline, and high levels of total blood cholesterol.

78. In 1971, T. Theodore Fujita created the Fujita Scale (or F Scale), which uses ratings from 0 to 5 to classify tornadoes, based on the damage that wind intensity causes to structures as the tornado passes through. This scale was updated by meteorologists and wind engineers working for the National Oceanic and Atmospheric Administration (NOAA) and implemented in February 2007. This new scale is called the Enhanced F Scale. An EF-0 tornado has wind speeds between 65 and 85 mph, inclusive. The winds in an EF-1 tornado range from 86 to 110 mph. In an EF-2 tornado, winds are from 111 to 135 mph. An EF-3 tornado has wind speeds ranging from 136 to 165 mph. Wind speeds in an EF-4 tornado are clocked at 166 to 200 mph. The most violent tornadoes are ranked at EF-5, with wind speeds of at least 201 mph. (Source: Storm Prediction Center, NOAA)

Letting \( y \) represent a tornado's wind speed, write a series of six inequalities that describe the wind speed ranges corresponding to each Enhanced Fujita Scale rank.

79. Twice a number, increased by one, is between negative five and seven. Find all such numbers.

80. Half a number, decreased by four, is between two and three. Find all such numbers.

81. The temperatures in Ohio range from \(-39\)°C to \(45\)°C. Use a compound inequality to convert these temperatures to Fahrenheit temperatures. (Hint: Use \( C = \frac{5}{9}(F - 32) \).)

82. Mario Lipco has scores of 85, 95, and 92 on his algebra tests. Use a compound inequality to find the range of scores he can make on his final exam in order to receive an A in the course. The final exam counts as three tests, and an A is received if the final course average is from 90 to 100. (Hint: The average of a list of numbers is their sum divided by the number of numbers in the list.)

**REVIEW AND PREVIEW**

Evaluate the following. See Section 1.4.

83. \( (2)^3 \)

84. \( (3)^3 \)

85. \( (1)^{12} \)

86. \( 0^7 \)

87. \( \left( \frac{4}{7} \right)^2 \)

88. \( \left( \frac{2}{3} \right)^3 \)
This broken line graph shows the average annual per person expenditure on newspapers for the given years. Use this graph for Exercises 89 through 92. (Source: Veronis Suhler Stevenson)

94. Bunnie Supplies manufactures plastic Easter eggs that open. The company has determined that if the circumference of the opening of each part of the egg is in the interval $118 \leq C \leq 122$ millimeters, the eggs will open and close comfortably. Use a compound inequality and find the corresponding interval for diameters of these openings. (Round to 2 decimal places.)

For Exercises 95 through 98, see the example below.

Solve each inequality. Graph the solution set and write it in interval notation.

95. $x(x - 6) > x^2 - 5x + 6$

96. $x(x - 3) \leq x^2 - 5x - 8$

97. $x^2 + 6x - 10 < x(x - 10)$

98. $x^2 - 4x + 8 < x(x + 8)$
THE BIGGER PICTURE  SIMPLIFYING EXPRESSIONS AND SOLVING EQUATIONS AND INEQUALITIES

Now we continue our outline from Sections 1.7 and 2.3. Although suggestions are given, this outline should be in your own words. Once you complete this new portion, try the exercises to the right.

I. Simplifying Expressions
   A. Real Numbers
      1. Add (Section 1.5)
      2. Subtract (Section 1.6)
      3. Multiply or Divide (Section 1.7)

II. Solving Equations
    A. Linear Equations (Section 2.3)

III. Solving Inequalities
    A. Linear Inequalities: Same as linear equations, except there are inequality symbols, ≤, <, ≥, >. Remember, if you multiply or divide by a negative number, then reverse the direction of the inequality symbol.

\[-4x - 11 \leq 1\]
\[\frac{-4x}{-4} \geq \frac{12}{-4}\]
\[x \geq -3\]

Solve each equation or inequality. Write inequality solutions in interval notation.

1. \(-5x = 15\)
2. \(-5x > 15\)
3. \(9y - 14 = -12\)
4. \(9x - 3 = 5x - 4\)
5. \(4(x - 2) \leq 5x + 7\)
6. \(5(4x - 1) = 2(10x - 1)\)
7. \(-5.4 = 0.6x - 9.6\)
8. \(\frac{1}{3}(x - 4) < \frac{1}{4}(x + 7)\)
9. \(3y - 5(y - 4) = -2(y - 10)\)
10. \(\frac{7(x - 1)}{3} = \frac{2(x + 1)}{5}\)

CHAPTER 2 GROUP ACTIVITY

Investigating Averages

Sections 2.1–2.8

Materials:
- small rubber ball or crumpled paper ball
- bucket or waste can

This activity may be completed by working in groups or individually.

1. Try shooting the ball into the bucket or waste can 5 times. Record your results below.

<table>
<thead>
<tr>
<th>Shots Made</th>
<th>Shots Missed</th>
</tr>
</thead>
</table>

2. Find your shooting percent for the 5 shots (that is, the percent of the shots you actually made out of the number you tried).

3. Suppose you are going to try an additional 5 shots. How many of the next 5 shots will you have to make to have a 50% shooting percent for all 10 shots? An 80% shooting percent?

4. Did you solve an equation in Question 3? If so, explain what you did. If not, explain how you could use an equation to find the answers.

5. Now suppose you are going to try an additional 22 shots. How many of the next 22 shots will you have to make to have at least a 50% shooting percent for all 27 shots? At least a 70% shooting percent?

6. Choose one of the sports played at your college that is currently in season. How many regular-season games are scheduled? What is the team’s current percent of games won?

7. Suppose the team has a goal of finishing the season with a winning percent better than 110% of their current wins. At least how many of the remaining games must they win to achieve their goal?
CHAPTER 2 VOCABULARY CHECK

Fill in each blank with one of the words or phrases listed below.

like terms numerical coefficient linear inequality in one variable
equivalent equations formula compound inequalities
linear equation in one variable

1. Terms with the same variables raised to exactly the same powers are called _________________.
2. A __________________ can be written in the form \( ax + b = c \).
3. Equations that have the same solution are called _________________.
4. Inequalities containing two inequality symbols are called _________________.
5. An equation that describes a known relationship among quantities is called a _________________.
6. A __________________ can be written in the form \( ax + b < c \), (or \( > \), \( \leq \), \( \geq \)).
7. The __________________ of a term is its numerical factor.

Helpful Hint

Are you preparing for your test? Don’t forget to take the Chapter 2 Test on page 162. Then check your answers at the back of the text and use the Chapter Test Prep Video CD to see the fully worked-out solutions to any of the exercises you want to review.

CHAPTER 2 HIGHLIGHTS

DEFINITIONS AND CONCEPTS

SECTION 2.1 SIMPLIFYING ALGEBRAIC EXPRESSIONS

The **numerical coefficient** of a **term** is its numerical factor.

Terms with the same variables raised to exactly the same powers are **like terms**.

To **combine like terms**, add the numerical coefficients and multiply the result by the common variable factor.

To remove parentheses, apply the distributive property.

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-7y)</td>
<td>(-7)</td>
</tr>
<tr>
<td>(x)</td>
<td>(1)</td>
</tr>
<tr>
<td>(1/5a^2b)</td>
<td>(1/5)</td>
</tr>
</tbody>
</table>

**Like Terms**

- \(12x, -x\)
- \(-2xy, 5yx\)
- \(7a^2b, -2ab^2\)

**Unlike Terms**

- \(9y + 3y = 12y\)
- \(-4z^2 + 5z^2 - 6z^2 = -5z^2\)
- \(-4(x + 7) + 10(3x - 1)\)
  - \(= -4x - 28 + 30x - 10\)
  - \(= 26x - 38\)
### DEFINITIONS AND CONCEPTS

**SECTION 2.2  THE ADDITION AND MULTIPLICATION PROPERTIES OF EQUALITY**

A **linear equation in one variable** can be written in the form \( ax + b = c \) where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

**Equivalent equations** are equations that have the same solution.

**Addition Property of Equality**
Adding the same number to or subtracting the same number from both sides of an equation does not change its solution.

**Multiplication Property of Equality**
Multiplying both sides or dividing both sides of an equation by the same nonzero number does not change its solution.

### EXAMPLES

**Linear Equations**

\[-3x + 7 = 2\]
\[3(x - 1) = -8(x + 5) + 4\]

\[x - 7 = 10 \text{ and } x = 17\]

are equivalent equations.

\[y + 9 = 3\]
\[y + 9 - 9 = 3 - 9\]
\[y = -6\]

\[\frac{2}{3}a = 18\]
\[\frac{2}{3} \left( \frac{2}{3}a \right) = \frac{2}{2} (18)\]
\[a = 27\]

### SECTIONS 2.2 & 2.3 SOLVING LINEAR EQUATIONS

**To Solve Linear Equations**

1. Clear the equation of fractions.
2. Remove any grouping symbols such as parentheses.
3. Simplify each side by combining like terms.
4. Write variable terms on one side and numbers on the other side using the addition property of equality.
5. Get the variable alone using the multiplication property of equality.
6. Check by substituting in the original equation.

Solve:

\[\frac{5(-2x + 9)}{6} + 3 = \frac{1}{2}\]

1. \[6 \cdot \frac{5(-2x + 9)}{6} + 6 \cdot 3 = 6 \cdot \frac{1}{2}\]
   \[5(-2x + 9) + 18 = 3\]

2. \[-10x + 45 + 18 = 3\]  
   **Distributive property**
3. \[-10x + 63 = 3\]  
   **Combine like terms.**
4. \[-10x + 63 - 63 = 3 - 63\]
   \[-10x = -60\]
   **Subtract 63.**
5. \[-\frac{10x}{-10} = \frac{-60}{-10}\]
   \[x = 6\]
   **Divide by \(-10.\)**

6. \[\frac{5(-2x + 9)}{6} + 3 = \frac{1}{2}\]
\[\frac{5(-2 \cdot 6 + 9)}{6} + 3 = \frac{1}{2}\]
\[\frac{5(-3)}{6} + 3 = \frac{1}{2}\]
\[-\frac{5}{2} + 6 = \frac{1}{2}\]
\[-\frac{1}{2} = \frac{1}{2}\]  
**True**
### DEFINITIONS AND CONCEPTS

#### Problem-Solving Steps

1. **UNDERSTAND** the problem.
2. **TRANSLATE** the problem.
3. **SOLVE**.
4. **INTERPRET** the results.

#### EXAMPLES

**SECTION 2.4 AN INTRODUCTION TO PROBLEM SOLVING**

The height of the Hudson volcano in Chili is twice the height of the Kiska volcano in the Aleutian Islands. If the sum of their heights is 12,870 feet, find the height of each.

1. Read and reread the problem. Guess a solution and check your guess.
   
   Let \( x \) be the height of the Kiska volcano. Then \( 2x \) is the height of the Hudson volcano.

   \[
   \begin{align*}
   \text{Kiska} & : x \\
   \text{Hudson} & : 2x \\
   \text{Height of added} & \quad \text{is} \quad \text{Height of Kiska to Hudson} \\
   \text{to} & = \quad 12,870 \\
   \text{Translate:} & \quad x + 2x = 12,870
   \end{align*}
   \]

2. **In words:**

   - If miles and miles per hour in the formula find \( t \).
   - Let \( r = 52 \) and \( d = 182 \).
   - The time is 3.5 hours.

   
   \[
   \begin{align*}
   \frac{d}{2} & = l \\
   P & = 2l + 2w \\
   P - 2w & = 2l + 2w - 2w \quad \text{Subtract} \ 2w. \\
   P - 2w & = 2l \quad \text{Divide by} \ 2. \\
   \frac{P - 2w}{2} & = l \quad \text{Simplify.}
   \end{align*}
   \]

3. **Check:** If \( x \) is 4290 then \( 2x \) is \( 2(4290) \) or 8580. Their sum is \( 4290 + 8580 \) or 12,870, the required amount.

   **State:** Kiska volcano is 4290 feet high and Hudson volcano is 8580 feet high.

**SECTION 2.5 FORMULAS AND PROBLEM SOLVING**

An equation that describes a known relationship among quantities is called a **formula**.

**To solve a formula for a specified variable**, use the same steps as for solving a linear equation. Treat the specified variable as the only variable of the equation.

If all values for the variables in a formula are known except for one, this unknown value may be found by substituting in the known values and solving.

**Formulas**

\[
A = lw \quad (\text{area of a rectangle}) \\
I = PRT \quad (\text{simple interest})
\]

Solve: \( P = 2l + 2w \) for \( l \).

\[
\begin{align*}
P & = 2l + 2w \\
P - 2w & = 2l + 2w - 2w \quad \text{Subtract} \ 2w. \\
P - 2w & = 2l \quad \text{Divide by} \ 2. \\
\frac{P - 2w}{2} & = l \quad \text{Simplify.}
\end{align*}
\]

If \( d = 182 \) miles and \( r = 52 \) miles per hour in the formula \( d = r \cdot t \), find \( t \).

\[
\begin{align*}
d & = r \cdot t \\
182 & = 52 \cdot t \quad \text{Let} \ d = 182 \text{ and} \ r = 52. \\
3.5 & = t
\end{align*}
\]

The time is 3.5 hours.
**DEFINITIONS AND CONCEPTS**

**EXAMPLES**

**SECTION 2.6 PERCENT AND MIXTURE PROBLEM SOLVING**

Use the same problem-solving steps to solve a problem containing percents.

1. **UNDERSTAND.**

2. **TRANSLATE.**

3. **SOLVE.**

4. **INTERPRET.**

32% of what number is 36.8?

1. **Read and reread. Propose a solution and check.**
   Let \( x \) = the unknown number.

2. **Translate.**
   \[
   \frac{32\%}{x} = 36.8
   \]
   Simplify.

3. **Solve:**
   \[
   \frac{32\%}{x} = 36.8 \\
   0.32x = 36.8 \\
   \frac{0.32x}{0.32} = \frac{36.8}{0.32} \\
   x = 115
   \]
   Simplify.

4. **Check, then state:** 32% of 115 is 36.8.

How many liters of a 20% acid solution must be mixed with a 50% acid solution in order to obtain 12 liters of a 30% solution?

1. **Read and reread. Guess a solution and check.**
   Let \( x \) = number of liters of 20% solution.
   Then \( 12 - x \) = number of liters of 50% solution.

2. **Translate:**
   \[
   \begin{array}{c|c|c}
   \text{No. of Liters} & \text{Acid Strength} & \text{Amount of Acid} \\
   \hline
   20\% \text{ Solution} & x & 0.20x \\
   50\% \text{ Solution} & 12 - x & 0.50(12 - x) \\
   30\% \text{ Solution} & 12 & 0.30(12) \\
   \end{array}
   \]

   In words:
   \[
   \text{acid in 20\% solution} + \text{acid in 50\% solution} = \text{acid in 30\% solution}
   \]

   Translate:
   \[
   0.20x + 0.50(12 - x) = 0.30(12)
   \]

3. **Solve:**
   \[
   0.20x + 0.50(12 - x) = 0.30(12) \\
   0.20x + 6 - 0.50x = 3.6 \\
   -0.30x + 6 = 3.6 \\
   -0.30x = -2.4 \\
   \frac{-0.30x}{-0.30} = \frac{-2.4}{-0.30} \\
   x = 8
   \]

4. **Check, then state:**
   If 8 liters of a 20% acid solution are mixed with \( 12 - 8 \) or 4 liters of a 50% acid solution, the result is 12 liters of a 30% solution.
1. **UNDERSTAND.**

2. **TRANSLATE.**

3. **SOLVE.**

4. **INTERPRET.**

**DEFINITIONS AND CONCEPTS**

**EXAMPLES**

**SECTION 2.7 FURTHER PROBLEM SOLVING**

**Problem-Solving Steps**

A collection of dimes and quarters has a total value of $19.55. If there are three times as many quarters as dimes, find the number of quarters.

1. **Read and reread. Propose a solution and check.**
   - Let \( x \) = number of dimes and 
     \( 3x \) = number of quarters.

2. **In words:** value of dimes + value of quarters = 19.55
   - Translate: 
     \[ 0.10x + 0.25(3x) = 19.55 \]

3. **Solve:**
   - Multiply.
   - Add like terms.
   - Divide by 0.85.

4. **Check, then state.**
   - The number of dimes is 23 and the number of quarters is \( 3(23) \) or 69. The total value of this money is 0.10(23) + 0.25(69) = 19.55, so our result checks.
   - The number of quarters is 69.

**SECTION 2.8 SOLVING LINEAR INEQUALITIES**

A **linear inequality in one variable** is an inequality that can be written in one of the forms:

\[ ax + b < c \]
\[ ax + b > c \]
where \( a, b, \) and \( c \) are real numbers and \( a \) is not 0.

**Addition Property of Inequality**

Adding the same number to or subtracting the same number from both sides of an inequality does not change the solutions.

**Multiplication Property of Inequality**

Multiplying or dividing both sides of an inequality by the same **negative number and reversing the direction of the inequality sign** does not change its solutions.
### DEFINITIONS AND CONCEPTS

**To Solve Linear Inequalities**
1. Clear the equation of fractions.
2. Remove grouping symbols.
3. Simplify each side by combining like terms.
4. Write variable terms on one side and numbers on the other side using the addition property of inequality.
5. Get the variable alone using the multiplication property of inequality.

Inequalities containing two inequality symbols are called **compound inequalities**.

To solve a compound inequality, isolate the variable in the middle part of the inequality. Perform the same operation to all three parts of the inequality: left, middle, right.

### CHAPTER 2 REVIEW

**2.1** Simplify the following expressions.

1. \(5x - x + 2x\)
2. \(0.2z - 4.6x - 7.4z\)
3. \(\frac{1}{2}x + 3 + \frac{7}{5}x - 5\)
4. \(\frac{4}{5}y + 1 + \frac{6}{3}y + 2\)
5. \(2(n - 4) + n - 10\)
6. \(3(w + 2) - (12 - w)\)
7. Subtract 7x - 2 from x + 5.
8. Subtract 1.4y - 3 from y - 0.7.

Write each of the following as algebraic expressions.

9. Three times a number decreased by 7
10. Twice the sum of a number and 2.8 added to 3 times the number

**2.2** Solve each equation.

11. \(8x + 4 = 9x\)
12. \(5y - 3 = 6y\)
13. \(\frac{2}{7}x + \frac{5}{7}x = 6\)
14. \(3x - 5 = 4x + 1\)
15. \(2x - 6 = x - 6\)
16. \(4(x + 3) = 3(1 + x)\)
17. \(6(3 + n) = 5(n - 1)\)
18. \(5(2 + x) - 3(3x + 2) = -5(x + 6) + 2\)

Use the addition property to fill in the blank so that the middle equation simplifies to the last equation.

19. \(x - 5 = \text{Blank} + 3\)
20. \(x + 9 = \text{Blank} - 2\)
21. The sum of two numbers is 10. If one number is \(x\), express the other number in terms of \(x\).
   a. \(x - 10\)
   b. \(10 - x\)
   c. \(10 + x\)
   d. \(10x\)
22. Mandy is 5 inches taller than Melissa. If \(x\) inches represents the height of Mandy, express Melissa’s height in terms of \(x\).
   a. \(x - 5\)
   b. \(5 - x\)
   c. \(5 + x\)
   d. \(5x\)
23. If one angle measures \( x^\circ \), express the measure of its complement in terms of \( x \).
   a. \((180 - x)^\circ\)
   b. \((90 - x)^\circ\)
   c. \((x - 180)^\circ\)
   d. \((x - 90)^\circ\)

24. If one angle measures \((x + 5)^\circ\), express the measure of its supplement in terms of \( x \).
   a. \((185 + x)^\circ\)
   b. \((95 + x)^\circ\)
   c. \((175 - x)^\circ\)
   d. \((x - 170)^\circ\)

Solve each equation.
25. \( \frac{3}{4}x = -9 \)
26. \( \frac{x}{6} = \frac{2}{3} \)
27. \(-5x = 0 \)
28. \(-8 = 7 \)
29. \(0.2x = 0.15 \)
30. \(-\frac{x}{3} = 1 \)
31. \(-3x + 1 = 19 \)
32. \(5x + 25 = 20 \)
33. \(7(x - 1) + 9 = 5x \)
34. \(2x - 6 = 5x - 3 \)
35. \(-5x + \frac{3}{7} = \frac{10}{7} \)
36. \(5x + x = 9 + 4x - 1 + 6 \)
37. Write the sum of three consecutive integers as an expression in \( x \). Let \( x \) be the first integer.
38. Write the sum of the first and fourth of four consecutive even integers. Let \( x \) be the first even integer.

(2.3) Solve each equation.
39. \( \frac{5}{3}x + 4 = \frac{2}{3}x \)
40. \( \frac{7}{8}x + 1 = \frac{5}{8}x \)
41. \(-5x + 1 = -7x + 5 \)
42. \(-4(2x + 1) = -5x + 5 \)
43. \(-6(2x - 5) = -3(9 + 4x) \)
44. \(3(8y - 1) = 6(5 + 4y) \)
45. \(\frac{3(2 - z)}{5} = z \)
46. \(4(n + 2) = -n - 5 \)
47. \(0.5(2n - 3) - 0.1 = 0.4(6 + 2n) \)
48. \(-9 - 5a = 3(6a - 1) \)
49. \(\frac{5(x + 1)}{6} = 2c - 3 \)
50. \(\frac{2(8 - a)}{3} = 4 - 4a \)
51. \(200(70x - 3560) = -179(150x - 19,300) \)
52. \(1.72y - 0.04y = 0.42 \)

(2.4) Solve each of the following.
53. The height of the Washington Monument is 50.5 inches more than 10 times the length of a side of its square base. If the sum of these two dimensions is 7327 inches, find the height of the Washington Monument. (Source: National Park Service)
54. A 12-foot board is to be divided into two pieces so that one piece is twice as long as the other. If \( x \) represents the length of the shorter piece, find the length of each piece.

55. In a recent year, Kellogg Company acquired Keebler Foods Company. After the merger, the total number of Kellogg and Keebler manufacturing plants was 53. The number of Kellogg plants was one less than twice the number of Keebler plants. How many of each type of plant were there? (Source: Kellogg Company 2000 Annual Report)
56. Find three consecutive integers whose sum is \(-114\).
57. The quotient of a number and 3 is the same as the difference of the number and two. Find the number.
58. Double the sum of a number and 6 is the opposite of the number. Find the number.

(2.5) Substitute the given values into the given formulas and solve for the unknown variable.
59. \( P = 2l + 2w; \quad P = 46, l = 14 \)
60. \( V = \text{lwh}; \quad V = 192, l = 8, w = 6 \)

Solve each equation for the indicated variable.
61. \( y = mx + b \) for \( m \)
62. \( r = \text{vst} - 5 \) for \( s \)
63. \( 2y - 5x = 7 \) for \( x \)
64. \( 3x - 6y = -2 \) for \( y \)

65. \( C = \pi D \) for \( \pi \)
66. \( C = 2\pi r \) for \( \pi \)
67. A swimming pool holds 900 cubic meters of water. If its length is 20 meters and its height is 3 meters, find its width.

68. The perimeter of a rectangular billboard is 60 feet and has a length 6 feet longer than its width. Find the dimensions of the billboard.
69. A charity 10K race is given annually to benefit a local hospice organization. How long will it take to run/walk a 10K race (10 kilometers or 10,000 meters) if your average pace is 125 meters per minute? Give your time in hours and minutes.

70. On April 28, 2001, the highest temperature recorded in the United States was 104°F, which occurred in Death Valley, California. Convert this temperature to degrees Celsius. (Source: National Weather Service)

(2.6) Find each of the following.

71. The number 9 is what percent of 45?
72. The number 59.5 is what percent of 85?
73. The number 137.5 is 125% of what number?
74. The number 768 is 60% of what number?
75. The price of a small diamond ring was recently increased by 11%. If the ring originally cost $1900, find the mark-up and the new price of the ring.

76. A recent survey found that 66.9% of Americans use the Internet. If a city has a population of 76,000 how many people in that city would you expect to use the Internet? (Source: UCLA Center for Communication Policy)

77. Thirty gallons of a 20% acid solution is needed for an experiment. Only 40% and 10% acid solutions are available. How much of each should be mixed to form the needed solution?

78. The ACT Assessment is a college entrance exam taken by about 60% of college-bound students. The national average score was 20.7 in 1993 and rose to 21.0 in 2001. Find the percent increase. (Round to the nearest hundredth of a percent.)

(2.7) Solve.

81. If a cell-phone service has an estimated 4600 customers who use their cell phones while driving, how many of these customers would you expect to have cut someone off while driving and talking on their cell phones?

82. Do the percents in the graph have a sum of 100%? Why or why not?

83. In 2005, Lance Armstrong incredibly won his seventh Tour de France, the first man in history to win more than five Tour de France Championships. Suppose he rides a bicycle up a category 2 climb at 10 km/hr and rides down the same distance at a speed of 50 km/hr. Find the distance traveled if the total time on the mountain was 3 hours.

84. A $50,000 retirement pension is to be invested into two accounts: a money market fund that pays 8.5% and a certificate of deposit that pays 10.5%. How much should be invested at each rate in order to provide a yearly interest income of $4550?

85. A pay phone is holding its maximum number of 500 coins consisting of nickels, dimes, and quarters. The number of quarters is twice the number of dimes. If the value of all the coins is $88.00, how many nickels were in the pay phone?

86. How long will it take an Amtrak passenger train to catch up to a freight train if their speeds are 60 and 45 mph and the freight train had an hour and a half head start?

(2.8) Solve and graph the solution of each of the following inequalities.

87. $x > 0$
88. $x \leq -2$
89. $0.5 \leq y < 1.5$
90. $-1 < x < 1$
91. $-3x > 12$
92. $-2x \geq -20$
93. $x + 4 \geq 6x - 16$
94. $5x - 7 > 8x + 5$
95. $-3 < 4x - 1 < 2$
96. $2 \leq 3x - 4 < 6$
97. $4(2x - 5) \leq 5x - 1$
98. $-2(x - 5) > 2(3x - 2)$
99. Tina earns $175 per week plus a 5% commission on all her sales. Find the minimum amount of sales to ensure that she earns at least $300 per week.

100. Ellen Catarella shot rounds of 76, 82, and 79 golfing. What must she shoot on her next round so that her average will be below 80?

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79. What percent of motorists who use a cell phone while driving have almost hit another car?

80. What is the most common effect of cell phone use on driving?

The graph below shows the percent(s) of cell phone users who have engaged in various behaviors while driving and talking on their cell phones. Use this graph to answer Exercises 79 through 82.

The graph below shows the percent(s) of cell phone users who have engaged in various behaviors while driving and talking on their cell phones. Use this graph to answer Exercises 79 through 82.
MIXED REVIEW

Solve each equation.

101. \(6x + 2x - 1 = 5x + 11\)
102. \(2(3y - 4) = 6 + 7y\)
103. \(4(3 - a) - (6a + 9) = -12a\)
104. \(\frac{x}{3} = 2\)
105. \(2(y + 5) = 2y + 10\)
106. \(7x - 3x + 2 = 2(2x - 1)\)

Solve.

107. The sum of six and twice a number is equal to seven less than the number. Find the number.

108. A 23-inch piece of string is to be cut into two pieces so that the length of the longer piece is three more than four times the shorter piece. If \(x\) represents the length of the shorter piece, find the lengths of both pieces.

109. Solve for the specified variable.

110. What number is 26% of 85?

111. The number 72 is 45% of what number?

112. A company recently increased their number of employees from 235 to 282. Find the percent increase.

Solve each inequality. Graph the solution set.

113. \(-3(1 + 2x) + x \geq -(3 - x)\)
114. \(-5x < 20\)
115. \(-3(1 + 2x) + x \geq -(3 - x)\)

CHAPTER 2 TEST

Simplify each of the following expressions.

1. \(2y - 6 - y - 4\)
2. \(2.7x + 6.1 + 3.2x - 4.9\)
3. \(3(x - 2) - 3(2x - 6)\)
4. \(7 + 2(3y - 3)\)

Solve each of the following equations.

5. \(\frac{4}{5}x = 4\)
6. \(4(x - 5) = -(4 - 2n)\)
7. \(5y - 7 + y = -(y + 3y)\)
8. \(4z + 1 - z = 1 + z\)
9. \(\frac{2x + 6}{3} = x - 5\)
10. \(\frac{1}{2} - x + \frac{3}{2} = x - 4\)
11. \(-0.3(x - 4) + x = 0.5(3 - x)\)
12. \(-4(a + 1) - 3a = -7(2a - 3)\)
13. \(-2(x - 3) = x + 5 - 3x\)

Solve each of the following applications.

14. A number increased by two-thirds of the number is 35. Find the number.
15. A gallon of water seal covers 200 square feet. How many gallons are needed to paint two coats of water seal on a deck that measures 20 feet by 35 feet?

16. Some states have a single area code for the entire state. Two such states have area codes where one is double the other. If the sum of these integers is 1203, find the two area codes. (Source: North American Numbering Plan Administration)

17. Sedric Angell invested an amount of money in Amoxil stock that earned an annual 10% return, and then he invested twice the original amount in IBM stock that earned an annual 12% return. If his total return from both investments was $2890, find how much he invested in each stock.

18. Two trains leave Los Angeles simultaneously traveling on the same track in opposite directions at speeds of 50 and 64 mph. How long will it take before they are 285 miles apart?

19. Find the value of \(x\) if \(y = -14, m = -2,\) and \(b = -2\) in the formula \(y = mx + b.\)

Solve each of the following equations for the indicated variable.

20. \(V = \pi r^2 h\) for \(h\)
21. \(3x - 4y = 10\) for \(y\)

Solve and graph each of the following inequalities.

22. \(3x - 5 \geq 7x + 3\)
23. \(x + 6 > 4x - 6\)
24. \(-2 < 3x + 1 < 8\)
25. \(\frac{2(5x + 1)}{3} > 2\)
CHAPTER 2 CUMULATIVE REVIEW

1. Given the set \( \{ -2, 0, \frac{1}{4}, -1.5, 112, -3, 11, \sqrt{2} \} \), list the numbers in this set that belong to the set of:
   a. Natural numbers
   b. Whole numbers
   c. Integers
   d. Rational numbers
   e. Irrational numbers
   f. Real numbers

2. Given the set \( \{ 7, 2, \frac{1}{5}, 0, \sqrt{3}, -185, 8 \} \), list the numbers in this set that belong to the set of:
   a. Natural numbers
   b. Whole numbers
   c. Integers
   d. Rational numbers
   e. Irrational numbers
   f. Real numbers

3. Find the absolute value of each number:
   a. \( |4| \)
   b. \( |-5| \)
   c. \( |0| \)
   d. \( |-\frac{1}{2}| \)
   e. \( |5.6| \)

4. Find the absolute value of each number:
   a. \( |5| \)
   b. \( |-8| \)
   c. \( |-\frac{2}{3}| \)

5. Write each of the following numbers as a product of primes.
   a. 40
   b. 63

6. Write each number as a product of primes.
   a. 44
   b. 90

7. Write \( \frac{2}{5} \) as an equivalent fraction with a denominator of 20.
8. Write \( \frac{2}{3} \) as an equivalent fraction with a denominator of 24.

9. Simplify \( 3[4 + 2(10 - 1)] \).
10. Simplify \( 5[16 - 4(2 + 1)] \).

11. Decide whether \( 2 \) is a solution of \( 3x + 10 = 8x \).
12. Decide whether \( 3 \) is a solution of \( 5x - 2 = 4x \).

Add.
13. \(-1 + (-2)\)
14. \((-2) + (-8)\)
15. \(-4 + 6\)
16. \(-3 + 10\)

17. Simplify each expression.
   a. \(-(-10)\)
   b. \(-\left( -\frac{1}{2} \right) \)
   c. \(-(-2x)\)
   d. \(-|-6|\)

18. Simplify each expression.
   a. \(-(-5)\)
   b. \(-\left( -\frac{2}{3} \right) \)
   c. \(-(-a)\)
   d. \(-|-3|\)

   a. \(5.3 - (-4.6)\)
   b. \(-\frac{3}{10} - \frac{5}{10}\)
   c. \(-\frac{2}{3} - \left( -\frac{4}{5} \right) \)

20. Subtract
   a. \(-2.7 - 8.4\)
   b. \(-\frac{4}{5} - \left( \frac{3}{5} \right) \)
   c. \(\frac{1}{4} - \left( -\frac{1}{2} \right) \)

21. Find each unknown complementary or supplementary angle.

22. Find each unknown complementary or supplementary angle.

23. Find each product.
   a. \((-1.2)(0.05)\)
   b. \(\frac{2}{3} \cdot \left( -\frac{7}{10} \right) \)
   c. \(-\frac{4}{5} \cdot (-20)\)

24. Find each product.
   a. \((4.5)(-0.08)\)
   b. \(-\frac{3}{4} \cdot \left( -\frac{8}{17} \right) \)

25. Find each quotient.
   a. \(-\frac{24}{-4}\)
   b. \(-\frac{36}{-3}\)
   c. \(\frac{2}{3} \div \left( -\frac{5}{4} \right) \)
   d. \(-\frac{3}{2} \div 9\)

26. Find each quotient.
   a. \(-\frac{32}{8}\)
   b. \(-\frac{108}{-12}\)
   c. \(-\frac{5}{7} \div \left( -\frac{9}{2} \right) \)

27. Use a commutative property to complete each statement.
   a. \(x + 5 = \) _____
   b. \(3 \cdot x = \) _____

28. Use a commutative property to complete each statement.
   a. \(y + 1 = \) _____
   b. \(y \cdot 4 = \) _____
29. Use the distributive property to write each sum as a product.
   \( a. \ 8 \cdot 2 + 8 \cdot x \)
   \( b. \ 7s + 7t \)
30. Use the distributive property to write each sum as a product.
   \( a. \ 4 \cdot y + 4 \cdot \frac{1}{3} \)
   \( b. \ 0.10x + 0.10y \)
31. Subtract \( 4x - 2 \) from \( 2x - 3 \).
32. Subtract \( 10x + 3 \) from \( -5x + 1 \).

**Solve.**
33. \( y + 0.6 = -1.0 \)
34. \( \frac{5}{6} + x = \frac{2}{3} \)
35. \( 7 = -5(2a - 1) - (-11a + 6) \)
36. \( -3x + 1 - (-4x - 6) = 10 \)
37. \( \frac{y}{7} = 20 \)
38. \( \frac{x}{4} = 18 \)
39. \( 4(2x - 3) + 7 = 3x + 5 \)
40. \( 6x + 5 = 4(x + 4) - 1 \)
41. Twice the sum of a number and 4 is the same as four times the number, decreased by 12. Find the number.
42. A number increased by 4 is the same as 3 times the number decreased by 8. Find the number.
43. Solve \( V = lwh \) for \( l \).
44. Solve \( C = 2\pi r \) for \( r \).
45. Solve \( x + 4 \leq -6 \) for \( x \). Graph the solution set and write it in interval notation.
46. Solve \( x - 3 > 2 \) for \( x \). Graph the solution set and write it in interval notation.