Online advertising is a way to promote services and products via the Internet. This fairly new way of advertising is quickly growing in popularity as more of the world is looking to the Internet for news and information. The broken-line graph below shows the yearly revenue generated by online advertising.

In Chapter 3’s Integrated Review, Exercise 16, you will have the opportunity to use a linear equation, generated by the years 2003–2010, to predict online advertising revenue.

In the previous chapter we learned to solve and graph the solutions of linear equations and inequalities in one variable. Now we define and present techniques for solving and graphing linear equations and inequalities in two variables.
3.1 READING GRAPHS AND THE RECTANGULAR COORDINATE SYSTEM

In today’s world, where the exchange of information must be fast and entertaining, graphs are becoming increasingly popular. They provide a quick way of making comparisons, drawing conclusions, and approximating quantities.

OBJECTIVE 1 Reading bar and line graphs. A bar graph consists of a series of bars arranged vertically or horizontally. The bar graph in Example 1 shows a comparison of worldwide Internet users by country. The names of the countries are listed vertically and a bar is shown for each country. Corresponding to the length of the bar for each country is a number along a horizontal axis. These horizontal numbers are number of Internet users in millions.

EXAMPLE 1

The following bar graph shows the estimated number of Internet users worldwide by country, as of a recent year.

a. Find the country that has the most Internet users and approximate the number of users.
b. How many more users are in the United States than in China?

Solution

a. Since these bars are arranged horizontally, we look for the longest bar, which is the bar representing the United States. To approximate the number associated with this country, we move from the right edge of this bar vertically downward to the Internet user axis. This country has approximately 198 million Internet users.
b. The United States has approximately 198 million Internet users. China has approximately 120 million Internet users. To find how many more users are in the United States, we subtract $198 - 120 = 78$ or 78 million more Internet users.

PRACTICE 1 Use the graph from Example 1 to answer the following.

a. Find the country shown with the fewest Internet users and approximate the number of users.
b. How many more users are in India than in Germany?

A line graph consists of a series of points connected by a line. The next graph is an example of a line graph. It is also sometimes called a broken line graph.
EXAMPLE 2  The line graph shows the relationship between time spent smoking a cigarette and pulse rate. Time is recorded along the horizontal axis in minutes, with 0 minutes being the moment a smoker lights a cigarette. Pulse is recorded along the vertical axis in heartbeats per minute.

**Example 2**

The line graph shows the relationship between time spent smoking a cigarette and pulse rate. Time is recorded along the horizontal axis in minutes, with 0 minutes being the moment a smoker lights a cigarette. Pulse is recorded along the vertical axis in heartbeats per minute.

**a.** What is the pulse rate 15 minutes after a cigarette is lit?

**b.** When is the pulse rate the lowest?

**c.** When does the pulse rate show the greatest change?

**Solution**

**a.** We locate the number 15 along the time axis and move vertically upward until the line is reached. From this point on the line, we move horizontally to the left until the pulse rate axis is reached. Reading the number of beats per minute, we find that the pulse rate is 80 beats per minute 15 minutes after a cigarette is lit.

**b.** We find the lowest point of the line graph, which represents the lowest pulse rate. From this point, we move vertically downward to the time axis. We find that the pulse rate is the lowest at -5 minutes, which means 5 minutes before lighting a cigarette.

**c.** The pulse rate shows the greatest change during the 5 minutes between 0 and 5. Notice that the line graph is steepest between 0 and 5 minutes.

**Practice 2** Use the graph from Example 2 to answer the following.

**a.** What is the pulse rate 40 minutes after lighting a cigarette?

**b.** What is the pulse rate when the cigarette is being lit?

**c.** When is the pulse rate the highest?
OBJECTIVE 2 ▶ Defining the rectangular coordinate system and plotting ordered pairs of numbers. Notice in the previous graph that there are two numbers associated with each point of the graph. For example, we discussed earlier that 15 minutes after lighting a cigarette, the pulse rate is 80 beats per minute. If we agree to write the time first and the pulse rate second, we can say there is a point on the graph corresponding to the ordered pair of numbers \((15, 80)\). A few more ordered pairs are listed alongside their corresponding points.

In general, we use this same ordered pair idea to describe the location of a point in a plane (such as a piece of paper). We start with a horizontal and a vertical axis. Each axis is a number line, and for the sake of consistency we construct our axes to intersect at the 0 coordinate of both. This point of intersection is called the origin. Notice that these two number lines or axes divide the plane into four regions called quadrants. The quadrants are usually numbered with Roman numerals as shown. The axes are not considered to be in any quadrant.

It is helpful to label axes, so we label the horizontal axis the \(x\)-axis and the vertical axis the \(y\)-axis. We call the system described above the rectangular coordinate system.

Just as with the pulse rate graph, we can then describe the locations of points by ordered pairs of numbers. We list the horizontal \(x\)-axis measurement first and the vertical \(y\)-axis measurement second.

To plot or graph the point corresponding to the ordered pair \((a, b)\) we start at the origin. We then move \(a\) units left or right (right if \(a\) is positive, left if \(a\) is negative). From there, we move \(b\) units up or down (up if \(b\) is positive, down if \(b\) is negative). For example, to plot the point corresponding to the ordered pair \((3, 2)\), we start at the origin, move 3 units right, and from there move 2 units up. (See the figure to the left.) The \(x\)-value, 3, is called the \(x\)-coordinate and the \(y\)-value, 2, is called the \(y\)-coordinate. From now on, we will call the point with coordinates \((3, 2)\) simply the point \((3, 2)\). The point \((-2, 5)\) is graphed to the left also.
Section 3.1  Reading Graphs and the Rectangular Coordinate System

Does the order in which the coordinates are listed matter? Yes! Notice that the point corresponding to the ordered pair \((2, 3)\) is in a different location than the point corresponding to \((3, 2)\). These two ordered pairs of numbers describe two different points of the plane.

Concept Check

Is the graph of the point \((-5, 1)\) in the same location as the graph of the point \((1, -5)\)? Explain.

Helpful Hint

Don’t forget that each ordered pair corresponds to exactly one point in the plane and that each point in the plane corresponds to exactly one ordered pair.

**EXAMPLE 3**  On a single coordinate system, plot each ordered pair. State in which quadrant, if any, each point lies.

\[
\begin{align*}
\text{a.} & \quad (5, 3) & \text{b.} & \quad (-5, 3) & \text{c.} & \quad (-2, -4) & \text{d.} & \quad (1, -2) \\
\text{e.} & \quad (0, 0) & \text{f.} & \quad (0, 2) & \text{g.} & \quad (-5, 0) & \text{h.} & \quad \left(0, -\frac{5}{2}\right)
\end{align*}
\]

**Solution**

Point \((5, 3)\) lies in quadrant I.
Point \((-5, 3)\) lies in quadrant II.
Point \((-2, -4)\) lies in quadrant III.
Point \((1, -2)\) lies in quadrant IV.

Points \((0, 0)\), \((0, 2)\), \((-5, 0)\), and \(0, -\frac{5}{2}\) lie on axes, so they are not in any quadrant.

From Example 3, notice that the \(y\)-coordinate of any point on the \(x\)-axis is 0. For example, the point \((-5, 0)\) lies on the \(x\)-axis. Also, the \(x\)-coordinate of any point on the \(y\)-axis is 0. For example, the point \((0, 2)\) lies on the \(y\)-axis.

**PRACTICE 3**  On a single coordinate system, plot each ordered pair. State in which quadrant, if any, each point lies.

\[
\begin{align*}
\text{a.} & \quad (4, -3) & \text{b.} & \quad (-3, 5) & \text{c.} & \quad (0, 4) & \text{d.} & \quad (-6, 1) \\
\text{e.} & \quad (-2, 0) & \text{f.} & \quad (5, 5) & \text{g.} & \quad \left(\frac{3}{2}, \frac{1}{2}\right) & \text{h.} & \quad (-4, -5)
\end{align*}
\]
CHAPTER 3 Graphs and Introduction to Functions

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OBJECTIVE 3 Graphing paired data. Data that can be represented as an ordered pair is called paired data. Many types of data collected from the real world are paired data. For instance, the annual measurement of a child’s height can be written as an ordered pair of the form (year, height in inches) and is paired data. The graph of paired data as points in the rectangular coordinate system is called a scatter diagram. Scatter diagrams can be used to look for patterns and trends in paired data.

EXAMPLE 4 The table gives the annual net sales for Wal-Mart Stores for the years shown. (Source: Wal-Mart Stores, Inc.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Wal-Mart Net Sales (in billions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>181</td>
</tr>
<tr>
<td>2001</td>
<td>204</td>
</tr>
<tr>
<td>2002</td>
<td>230</td>
</tr>
<tr>
<td>2003</td>
<td>256</td>
</tr>
<tr>
<td>2004</td>
<td>285</td>
</tr>
<tr>
<td>2005</td>
<td>312</td>
</tr>
<tr>
<td>2006</td>
<td>345</td>
</tr>
</tbody>
</table>

a. Write this paired data as a set of ordered pairs of the form (year, sales in billions of dollars).

b. Create a scatter diagram of the paired data.

c. What trend in the paired data does the scatter diagram show?

Solution


b. We begin by plotting the ordered pairs. Because the x-coordinate in each ordered pair is a year, we label the x-axis “Year” and mark the horizontal axis with the years given. Then we label the y-axis or vertical axis “Net Sales (in billions of dollars).” In this case it is convenient to mark the vertical axis in multiples of 20. Since no net sale is less than 180, we use the notation \( \uparrow \) to skip to 180, then proceed by multiples of 20.

c. The scatter diagram shows that Wal-Mart net sales steadily increased over the years 2000–2006.
Section 3.1 Reading Graphs and the Rectangular Coordinate System

The table gives the approximate annual number of wildfires (in the thousands) that have occurred in the United States for the years shown. (Source: National Interagency Fire Center)

<table>
<thead>
<tr>
<th>Year</th>
<th>Wildfires (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>92</td>
</tr>
<tr>
<td>2001</td>
<td>84</td>
</tr>
<tr>
<td>2002</td>
<td>73</td>
</tr>
<tr>
<td>2003</td>
<td>64</td>
</tr>
<tr>
<td>2004</td>
<td>65</td>
</tr>
<tr>
<td>2005</td>
<td>67</td>
</tr>
<tr>
<td>2006</td>
<td>96</td>
</tr>
</tbody>
</table>

**a.** Write this paired data as a set of ordered pairs of the form (year, number of wildfires in thousands).

**b.** Create a scatter diagram of the paired data.

**OBJECTIVE 4** Determining whether an ordered pair is a solution. Let’s see how we can use ordered pairs to record solutions of equations containing two variables. An equation in one variable such as $x + 1 = 5$ has one solution, which is 4: the number 4 is the value of the variable $x$ that makes the equation true.

An equation in two variables, such as $2x + y = 8$, has solutions consisting of two values, one for $x$ and one for $y$. For example, $x = 3$ and $y = 2$ is a solution of $2x + y = 8$ because, if $x$ is replaced with 3 and $y$ with 2, we get a true statement.

$$2x + y = 8$$
$$2(3) + 2 = 8$$
$$6 + 2 = 8$$
$$8 = 8 \quad \text{True}$$

The solution $x = 3$ and $y = 2$ can be written as $(3, 2)$, an ordered pair of numbers. The first number, 3, is the $x$-value and the second number, 2, is the $y$-value.

In general, an ordered pair is a solution of an equation in two variables if replacing the variables by the values of the ordered pair results in a true statement.

**EXAMPLE 5** Determine whether each ordered pair is a solution of the equation $x - 2y = 6$.

**a.** $(6, 0)$  
**b.** $(0, 3)$  
**c.** $\left(1, -\frac{5}{2}\right)$

**Solution**

**a.** Let $x = 6$ and $y = 0$ in the equation $x - 2y = 6$.

$$x - 2y = 6$$
$$6 - 2(0) = 6 \quad \text{Replace x with 6 and y with 0.}$$
$$6 - 0 = 6 \quad \text{Simplify.}$$
$$6 = 6 \quad \text{True}$$

$(6, 0)$ is a solution, since $6 = 6$ is a true statement.
b. Let \( x = 0 \) and \( y = 3 \).

\[
x - 2y = 6 \\
0 - 2(3) = 6 \\
0 - 6 = 6 \\
-6 = 6 \quad \text{False}
\]

\((0, 3)\) is not a solution, since \(-6 = 6\) is a false statement.

c. Let \( x = 1 \) and \( y = -\frac{5}{2} \) in the equation.

\[
x - 2y = 6 \\
1 - 2\left(-\frac{5}{2}\right) = 6 \\
1 + 5 = 6 \\
6 = 6 \quad \text{True}
\]

\(\left(1, -\frac{5}{2}\right)\) is a solution, since \(6 = 6\) is a true statement.

**Practice**

Determine whether each ordered pair is a solution of the equation \( x + 3y = 6 \).

- a. \((3, 1)\)
- b. \((6, 0)\)
- c. \((-2, \frac{2}{3})\)

**Objective 5** Completing ordered pair solutions. If one value of an ordered pair solution of an equation is known, the other value can be determined. To find the unknown value, replace one variable in the equation by its known value. Doing so results in an equation with just one variable that can be solved for the variable using the methods of Chapter 2.

**Example 6** Complete the following ordered pair solutions for the equation \(3x + y = 12\).

- a. \((0, \quad )\)
- b. \((\quad , 6)\)
- c. \((-1, \quad )\)

**Solution**

a. In the ordered pair \((0, \quad )\), the \(x\)-value is 0. Let \(x = 0\) in the equation and solve for \(y\).

\[
3x + y = 12 \\
3(0) + y = 12 \\
0 + y = 12 \\
y = 12
\]

The completed ordered pair is \((0, 12)\).

b. In the ordered pair \((\quad , 6)\), the \(y\)-value is 6. Let \(y = 6\) in the equation and solve for \(x\).

\[
3x + y = 12 \\
3x + 6 = 12 \quad \text{Replace } y \text{ with } 6. \\
3x = 6 \quad \text{Subtract 6 from both sides.} \\
x = 2 \quad \text{Divide both sides by 3.}
\]

The ordered pair is \((2, 6)\).
c. In the ordered pair \((-1, \phantom{0})\), the \(x\)-value is \(-1\). Let \(x = -1\) in the equation and solve for \(y\).

\[
3x + y = 12 \\
3(-1) + y = 12 \quad \text{Replace } x \text{ with } -1. \\
-3 + y = 12 \\
y = 15 \quad \text{Add 3 to both sides.}
\]

The ordered pair is \((-1, 15)\).

PRACTICE 6 Complete the following ordered pair solutions for the equation \(2x - y = 8\).

a. \((0, \phantom{0})\)  
   b. \((-4, 4)\)  
   c. \((-3, \phantom{0})\)

Solutions of equations in two variables can also be recorded in a **table of values**, as shown in the next example.

**EXAMPLE 7** Complete the table for the equation \(y = 3x\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (-1)</td>
<td>(-3)</td>
</tr>
<tr>
<td>b. (0)</td>
<td>(0)</td>
</tr>
<tr>
<td>c. (-9)</td>
<td></td>
</tr>
</tbody>
</table>

**Solution**

a. Replace \(x\) with \(-1\) in the equation and solve for \(y\).

\[y = 3x\]
\[y = 3(-1) \quad \text{Let } x = -1.\]
\[y = -3\]

The ordered pair is \((-1, -3)\).

b. Replace \(y\) with 0 in the equation and solve for \(x\).

\[y = 3x\]
\[0 = 3x \quad \text{Let } y = 0.\]
\[0 = x \quad \text{Divide both sides by 3.}\]

The ordered pair is \((0, 0)\).

c. Replace \(y\) with \(-9\) in the equation and solve for \(x\).

\[y = 3x\]
\[-9 = 3x \quad \text{Let } y = -9.\]
\[-3 = x \quad \text{Divide both sides by 3.}\]

The ordered pair is \((-3, -9)\). The completed table is shown to the left.

PRACTICE 7 Complete the table for the equation \(y = -4x\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (-2)</td>
<td>(8)</td>
</tr>
<tr>
<td>b. (-12)</td>
<td></td>
</tr>
<tr>
<td>c. (0)</td>
<td></td>
</tr>
</tbody>
</table>


### Example 8

Complete the table for the equation 

\[ y = \frac{1}{2}x - 5. \]

**Solution**

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
<th>c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = -2 )</td>
<td>( x = 0 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( y = \frac{1}{2}(-2) - 5 )</td>
<td>( y = \frac{1}{2}(0) - 5 )</td>
<td>( y = \frac{1}{2}(0) - 5 )</td>
</tr>
<tr>
<td>( y = -1 - 5 )</td>
<td>( y = 0 - 5 )</td>
<td>( 0 = \frac{1}{2}x - 5 )</td>
</tr>
<tr>
<td>( y = -6 )</td>
<td>( y = -5 )</td>
<td>Now, solve for ( x ).</td>
</tr>
</tbody>
</table>

Ordered Pairs: \((-2, -6)\) \((0, -5)\) \((10, 0)\)

The completed table is:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

### Practice 8

Compute the table for the equation \( y = \frac{1}{2}x - 2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

### Example 9

Finding the Value of a Computer

A computer was recently purchased for a small business for $2000. The business manager predicts that the computer will be used for 5 years and the value in dollars \( y \) of the computer in \( x \) years is \( y = -300x + 2000 \). Complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
</tr>
</tbody>
</table>

**Solution**

To find the value of \( y \) when \( x = 0 \), replace \( x \) with 0 in the equation. We use this same procedure to find \( y \) when \( x = 1 \) and when \( x = 2 \).

#### When \( x = 0 \),
- \( y = -300 \cdot 0 + 2000 \)
- \( y = 0 + 2000 \)
- \( y = 2000 \)

#### When \( x = 1 \),
- \( y = -300 \cdot 1 + 2000 \)
- \( y = -300 + 2000 \)
- \( y = 1700 \)

#### When \( x = 2 \),
- \( y = -300 \cdot 2 + 2000 \)
- \( y = -600 + 2000 \)
- \( y = 1400 \)

We have the ordered pairs \((0, 2000), (1, 1700), \) and \((2, 1400)\). This means that in 0 years the value of the computer is $2000, in 1 year the value of the computer is $1700, and in
2 years the value is $1400. To complete the table of values, we continue the procedure for \(x = 3, x = 4, \) and \(x = 5\).

<table>
<thead>
<tr>
<th>When (x = 3),</th>
<th>When (x = 4),</th>
<th>When (x = 5),</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = -300x + 2000)</td>
<td>(y = -300x + 2000)</td>
<td>(y = -300x + 2000)</td>
</tr>
<tr>
<td>(y = -900 + 2000)</td>
<td>(y = -1200 + 2000)</td>
<td>(y = -1500 + 2000)</td>
</tr>
<tr>
<td>(y = 1100)</td>
<td>(y = 800)</td>
<td>(y = 500)</td>
</tr>
</tbody>
</table>

The completed table is

\[
\begin{array}{ccccccc}
\text{ } & \text{0} & \text{1} & \text{2} & \text{3} & \text{4} & \text{5} \\
\text{ } & \text{y} & \text{2000} & \text{1700} & \text{1400} & \text{1100} & \text{800} & \text{500} \\
\end{array}
\]

**PRACTICE**

A college student purchased a used car for $12,000. The student predicted that she would need to use the car for four years and the value in dollars \(y\) of the car in \(x\) years is \(y = -1800x + 12,000\). Complete this table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>2000</td>
<td>1700</td>
<td>1400</td>
<td>1100</td>
<td>800</td>
</tr>
</tbody>
</table>

The ordered pair solutions recorded in the completed table for the example above are graphed below. Notice that the graph gives a visual picture of the decrease in value of the computer.

**VOCABULARY & READINESS CHECK**

Use the choices below to fill in each blank. The exercises below all have to do with the rectangular coordinate system.

- origin
- \(x\)-coordinate
- \(x\)-axis
- one
- four
- quadrants
- \(y\)-coordinate
- \(y\)-axis
- solution

1. The horizontal axis is called the ________.
2. The vertical axis is called the ________.
3. The intersection of the horizontal axis and the vertical axis is a point called the ________.
4. The axes divide the plane into regions, called _________. There are _________ of these regions.
5. In the ordered pair of numbers \((-2, 5)\), the number \(-2\) is called the ________ and the number 5 is called the ________.
6. Each ordered pair of numbers corresponds to ________ point in the plane.
7. An ordered pair is a _________ of an equation in two variables if replacing the variables by the coordinates of the ordered pair results in a true statement.
The following bar graph shows the top 10 tourist destinations and the number of tourists that visit each country per year. Use this graph to answer Exercises 1 through 6. See Example 1.

1. Which country shown is the most popular tourist destination?
2. Which country shown is the least popular tourist destination?
3. Which countries shown have more than 40 million tourists per year?
4. Which countries shown have between 40 and 50 million tourists per year?
5. Estimate the number of tourists per year whose destination is the United Kingdom.
6. Estimate the number of tourists per year whose destination is Turkey.

The following line graph shows the attendance at each Super Bowl game from 2000 through 2007. Use this graph to answer Exercises 7 through 10. See Example 2.


9. Find the year on the graph with the greatest Super Bowl attendance and approximate that attendance.
10. Find the year on the graph with the least Super Bowl attendance and approximate that attendance.

The line graph below shows the number of students per teacher in U.S. public elementary and secondary schools. Use this graph for Exercises 11 through 16. See Example 2.

11. Approximate the number of students per teacher in 2002.
12. Approximate the number of students per teacher in 2010.
13. Between what years shown did the greatest decrease in number of students per teacher occur?
14. What was the first year shown that the number of students per teacher fell below 17?
15. What was the first year shown that the number of students per teacher fell below 16?
16. Discuss any trends shown by this line graph.

Plot each ordered pair. State in which quadrant or on which axis each point lies. See Example 3.

17. a. (1, 5) b. (−5, −2) c. (−3, 0) d. (0, −1) e. (2, −4) f. \(-1, \frac{1}{2}\) g. (3, 7, 2, 2) h. \(\frac{1}{2}, -3\)
18. a. (2, 4) b. (0, 2) c. (−2, 1) d. (−3, −3) e. \(\frac{3}{4}, 0\) f. (5, −4) g. (−3, 4, 8) h. \(\frac{1}{3}, -5\)
Find the $x$- and $y$-coordinates of each labeled point. See Example 3.

19. $A$
20. $B$
21. $C$
22. $D$
23. $E$
24. $F$
25. $G$

26. $A$
27. $B$
28. $C$
29. $D$
30. $E$
31. $F$
32. $G$

Solve. See Example 4.

33. The table shows the number of regular-season NFL football games won by the winner of the Super Bowl for the years shown. (Source: National Football League)

<table>
<thead>
<tr>
<th>Year</th>
<th>Regular-Season Games Won by Super Bowl Winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>12</td>
</tr>
<tr>
<td>2003</td>
<td>14</td>
</tr>
<tr>
<td>2004</td>
<td>14</td>
</tr>
<tr>
<td>2005</td>
<td>11</td>
</tr>
<tr>
<td>2006</td>
<td>12</td>
</tr>
</tbody>
</table>

a. Write each paired data as an ordered pair of the form (year, games won).
b. Draw a grid such as the one in Example 4 and create a scatter diagram of the paired data.

c. What trend in the paired data does the scatter diagram show?

34. The table shows the average price of a gallon of regular unleaded gasoline (in dollars) for the years shown. (Source: Energy Information Administration)

<table>
<thead>
<tr>
<th>Year</th>
<th>Price per Gallon of Unleaded Gasoline (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1.38</td>
</tr>
<tr>
<td>2002</td>
<td>1.31</td>
</tr>
<tr>
<td>2003</td>
<td>1.52</td>
</tr>
<tr>
<td>2004</td>
<td>1.81</td>
</tr>
<tr>
<td>2005</td>
<td>2.24</td>
</tr>
<tr>
<td>2006</td>
<td>2.53</td>
</tr>
</tbody>
</table>

a. Write each paired data as an ordered pair of the form (year, gasoline price).
b. Draw a grid such as the one in Example 4 and create a scatter diagram of the paired data.

c. What trend in the paired data does the scatter diagram show?

35. The table shows the ethanol fuel production in the United States. (Source: Renewable Fuels Association; *some years projected)

<table>
<thead>
<tr>
<th>Year</th>
<th>Ethanol Fuel Production (in millions of gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1770</td>
</tr>
<tr>
<td>2003</td>
<td>2800</td>
</tr>
<tr>
<td>2005</td>
<td>3904</td>
</tr>
<tr>
<td>2007*</td>
<td>7500</td>
</tr>
<tr>
<td>2009*</td>
<td>10,800</td>
</tr>
</tbody>
</table>

a. Write each paired data as an ordered pair of the form (year, millions of gallons produced).
b. Draw a grid such as the one in Example 4 and create a scatter diagram of the paired data.
c. What trend in the paired data does the scatter diagram show?

36. The table shows the enrollment in college in the United States for the years shown. (Source: U.S. Department of Education)

<table>
<thead>
<tr>
<th>Year</th>
<th>Enrollment in College (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>8.6</td>
</tr>
<tr>
<td>1980</td>
<td>12.1</td>
</tr>
<tr>
<td>1990</td>
<td>13.8</td>
</tr>
<tr>
<td>2000</td>
<td>15.3</td>
</tr>
<tr>
<td>2010*</td>
<td>18.7</td>
</tr>
</tbody>
</table>

*projected

a. Write each paired data as an ordered pair of the form (year, college enrollment in millions).
b. Draw a grid such as the one in Example 4 and create a scatter diagram of the paired data.
c. What trend in the paired data does the scatter diagram show?

37. The table shows the distance from the equator (in miles) and the average annual snowfall (in inches) for each of eight

<table>
<thead>
<tr>
<th>Year</th>
<th>Distance from Equator (in miles)</th>
<th>Average Annual Snowfall (in inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 3  Graphs and Introduction to Functions

selected U.S. cities. (*Sources: National Climatic Data Center, Wake Forest University Albatross Project*

<table>
<thead>
<tr>
<th>City</th>
<th>Distance from Equator (in miles)</th>
<th>Average Annual Snowfall (in inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Atlanta, GA</td>
<td>2313</td>
<td>2</td>
</tr>
<tr>
<td>2. Austin, TX</td>
<td>2085</td>
<td>1</td>
</tr>
<tr>
<td>3. Baltimore, MD</td>
<td>2711</td>
<td>21</td>
</tr>
<tr>
<td>4. Chicago, IL</td>
<td>2869</td>
<td>39</td>
</tr>
<tr>
<td>5. Detroit, MI</td>
<td>2920</td>
<td>42</td>
</tr>
<tr>
<td>6. Juneau, AK</td>
<td>4038</td>
<td>99</td>
</tr>
<tr>
<td>7. Miami, FL</td>
<td>1783</td>
<td>0</td>
</tr>
<tr>
<td>8. Winston-Salem, NC</td>
<td>2493</td>
<td>9</td>
</tr>
</tbody>
</table>

**38.** The table shows the average farm size (in acres) in the United States during the years shown. (*Source: National Agricultural Statistics Service*)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Farm Size (in acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>438</td>
</tr>
<tr>
<td>2002</td>
<td>440</td>
</tr>
<tr>
<td>2003</td>
<td>441</td>
</tr>
<tr>
<td>2004</td>
<td>443</td>
</tr>
<tr>
<td>2005</td>
<td>445</td>
</tr>
<tr>
<td>2006</td>
<td>446</td>
</tr>
</tbody>
</table>

**39.** Write this paired data as a set of ordered pairs of the form (year, average farm size).

**40.** Create a scatter diagram of the paired data. Be sure to label the axes appropriately.

**41.** What trend in the paired data does the scatter diagram show?

**42.** The table shows the average farm size (in acres) in the United States during the years shown. (*Source: National Agricultural Statistics Service*)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Farm Size (in acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>438</td>
</tr>
<tr>
<td>2002</td>
<td>440</td>
</tr>
<tr>
<td>2003</td>
<td>441</td>
</tr>
<tr>
<td>2004</td>
<td>443</td>
</tr>
<tr>
<td>2005</td>
<td>445</td>
</tr>
<tr>
<td>2006</td>
<td>446</td>
</tr>
</tbody>
</table>

**43.** Write this paired data as a set of ordered pairs of the form (year, average farm size).

**44.** Create a scatter diagram of the paired data. Be sure to label the axes appropriately.

**45.** What trend in the paired data does the scatter diagram show?

**46.** The table shows the average farm size (in acres) in the United States during the years shown. (*Source: National Agricultural Statistics Service*)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Farm Size (in acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>438</td>
</tr>
<tr>
<td>2002</td>
<td>440</td>
</tr>
<tr>
<td>2003</td>
<td>441</td>
</tr>
<tr>
<td>2004</td>
<td>443</td>
</tr>
<tr>
<td>2005</td>
<td>445</td>
</tr>
<tr>
<td>2006</td>
<td>446</td>
</tr>
</tbody>
</table>

**47.** Write this paired data as a set of ordered pairs of the form (year, average farm size).

**48.** Create a scatter diagram of the paired data. Be sure to label the axes appropriately.

**49.** What trend in the paired data does the scatter diagram show?

**50.** The table shows the average farm size (in acres) in the United States during the years shown. (*Source: National Agricultural Statistics Service*)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Farm Size (in acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>438</td>
</tr>
<tr>
<td>2002</td>
<td>440</td>
</tr>
<tr>
<td>2003</td>
<td>441</td>
</tr>
<tr>
<td>2004</td>
<td>443</td>
</tr>
<tr>
<td>2005</td>
<td>445</td>
</tr>
<tr>
<td>2006</td>
<td>446</td>
</tr>
</tbody>
</table>

**51.** Write this paired data as a set of ordered pairs of the form (year, average farm size).

**52.** Create a scatter diagram of the paired data. Be sure to label the axes appropriately.

**53.** What trend in the paired data does the scatter diagram show?

**54.** The table shows the average farm size (in acres) in the United States during the years shown. (*Source: National Agricultural Statistics Service*)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Farm Size (in acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>438</td>
</tr>
<tr>
<td>2002</td>
<td>440</td>
</tr>
<tr>
<td>2003</td>
<td>441</td>
</tr>
<tr>
<td>2004</td>
<td>443</td>
</tr>
<tr>
<td>2005</td>
<td>445</td>
</tr>
<tr>
<td>2006</td>
<td>446</td>
</tr>
</tbody>
</table>

**55.** Write this paired data as a set of ordered pairs of the form (year, average farm size).

**56.** Create a scatter diagram of the paired data. Be sure to label the axes appropriately.

**57.** What trend in the paired data does the scatter diagram show?
Section 3.1 Reading Graphs and the Rectangular Coordinate System

57. \( y = 2x - 12 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

58. \( y = 5x + 10 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

59. \( 2x + 7y = 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

60. \( x - 6y = 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

**MIXED PRACTICE**

Complete the table of ordered pairs for each equation. Then plot the ordered pair solutions. See Examples 1 through 8.

61. \( x = -5y \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

62. \( y = -3x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

63. \( y = \frac{1}{3}x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

64. \( y = \frac{1}{2}x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Solve. See Example 9.**

65. The cost in dollars \( y \) of producing \( x \) computer desks is given by \( y = 80x + 5000 \).

a. Complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>

b. Find the number of computer desks that can be produced for \$8600. (Hint: Find \( x \) when \( y = 8600 \).)

66. The hourly wage \( y \) of an employee at a certain production company is given by \( y = 0.25x + 9 \) where \( x \) is the number of units produced by the employee in an hour.

a. Complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>9.25</td>
</tr>
<tr>
<td>5</td>
<td>11.5</td>
</tr>
<tr>
<td>10</td>
<td>12.5</td>
</tr>
</tbody>
</table>

b. Find the number of units that an employee must produce each hour to earn an hourly wage of \$12.25. (Hint: Find \( x \) when \( y = 12.25 \).)

67. The average amount of money \( y \) spent per person on recorded music from 2001 to 2005 is given by \( y = -2.35x + 55.92 \). In this equation, \( x \) represents the number of years after 2001. (Source: Veronis Suhler Stevenson)

a. Complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>53.57</td>
</tr>
<tr>
<td>3</td>
<td>51.22</td>
</tr>
<tr>
<td>5</td>
<td>48.88</td>
</tr>
</tbody>
</table>

b. Find the year in which the yearly average amount of money per person spent on recorded music was approximately \$46. (Hint: Find \( x \) when \( y = 46 \) and round to the nearest whole number.)

68. The amount \( y \) of land operated by farms in the United States (in million acres) from 2000 through 2006 is given by \( y = -2.18x + 944.68 \). In the equation, \( x \) represents the number of years after 2000. (Source: National Agricultural Statistics Service)

a. Complete the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>925.46</td>
</tr>
<tr>
<td>4</td>
<td>907.96</td>
</tr>
<tr>
<td>6</td>
<td>889.46</td>
</tr>
</tbody>
</table>

b. Find the year in which there were approximately 933 million acres of land operated by farms. (Hint: Find \( x \) when \( y = 933 \) and round to the nearest whole number.)

The graph below shows the number of Target stores for each year. Use this graph to answer Exercises 69 through 72.

69. The ordered pair (4, 1308) is a point of the graph. Write a sentence describing the meaning of this ordered pair.

70. The ordered pair (6, 1488) is a point of the graph. Write a sentence describing the meaning of this ordered pair.

71. Estimate the increase in Target stores for years 1, 2, and 3.

72. Use a straightedge or ruler and this graph to predict the number of Target stores in the year 2009.
73. When is the graph of the ordered pair \((a, b)\) the same as the graph of the ordered pair \((b, a)\)?

74. In your own words, describe how to plot an ordered pair.

**REVIEW AND PREVIEW**

Solve each equation for \(y\). See Section 2.5.

75. \(x + y = 5\)
76. \(x - y = 3\)
77. \(2x + 4y = 5\)
78. \(5x + 2y = 7\)
79. \(10x = 5y\)
80. \(4y = -8x\)
81. \(x - 3y = 6\)
82. \(2x - 9y = -20\)

**CONCEPT EXTENSIONS**

Answer each exercise with true or false.

83. Point \((-1, 5)\) lies in quadrant IV.
84. Point \((3, 0)\) lies on the \(y\)-axis.
85. For the point \(\left(\frac{1}{2}, 1.5\right)\), the first value, \(-\frac{1}{2}\), is the \(x\)-coordinate and the second value, 1.5, is the \(y\)-coordinate.
86. The ordered pair \(\left(\frac{2}{3}, 2\right)\) is a solution of \(2x - 3y = 6\).

For Exercises 87 through 91, fill in each blank with “0,” “positive,” or “negative.” For Exercises 92 and 93, fill in each blank with “\(x\)” or “\(y\).”

<table>
<thead>
<tr>
<th>Point</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>87. ((__, __))</td>
<td>quadrant III</td>
</tr>
<tr>
<td>88. ((__, __))</td>
<td>quadrant I</td>
</tr>
<tr>
<td>89. ((__, __))</td>
<td>quadrant IV</td>
</tr>
<tr>
<td>90. ((__, __))</td>
<td>quadrant II</td>
</tr>
<tr>
<td>91. ((__, __))</td>
<td>origin</td>
</tr>
<tr>
<td>92. ((\text{number, 0)})</td>
<td>(-\text{-axis})</td>
</tr>
<tr>
<td>93. ((0, \text{number)})</td>
<td>(+\text{-axis})</td>
</tr>
</tbody>
</table>

**STUDY SKILLS BUILDER**

Are You Satisfied with Your Performance in This Course Thus Far?

To see if there is room for improvement, answer these questions:

1. Am I attending all classes and arriving on time?
2. Am I working and checking my homework assignments on time?
3. Am I getting help (from my instructor or a campus learning resource lab) when I need it?

4. In addition to my instructor, am I using the text supplements that might help me?
5. Am I satisfied with my performance on quizzes and exams?

If you answered no to any of these questions, read or reread Section 1.1 for suggestions in these areas. Also, you might want to contact your instructor for additional feedback.
Section 3.2 Graphing Linear Equations

OBJECTIVES
1. Identify linear equations.
2. Graph a linear equation by finding and plotting ordered pair solutions.

OBJECTIVE 1 Identifying linear equations. In the previous section, we found that equations in two variables may have more than one solution. For example, both (6, 0) and (2, −2) are solutions of the equation \( x - 2y = 6 \). In fact, this equation has an infinite number of solutions. Other solutions include (0, −3), (4, −1), and (−2, −4). If we graph these solutions, notice that a pattern appears.

These solutions all appear to lie on the same line, which has been filled in below. It can be shown that every ordered pair solution of the equation corresponds to a point on this line, and every point on this line corresponds to an ordered pair solution. Thus, we say that this line is the graph of the equation \( x - 2y = 6 \).

The equation \( x - 2y = 6 \) is called a linear equation in two variables and the graph of every linear equation in two variables is a line.

Linear Equation in Two Variables
A linear equation in two variables is an equation that can be written in the form

\[ Ax + By = C \]

where \( A, B, \) and \( C \) are real numbers and \( A \) and \( B \) are not both 0. The graph of a linear equation in two variables is a straight line.

The form \( Ax + By = C \) is called standard form.

Helpful Hint
Notice in the form \( Ax + By = C \), the understood exponent on both \( x \) and \( y \) is 1.
Examples of Linear Equations in Two Variables

Before we graph linear equations in two variables, let’s practice identifying these equations.

**Example 1**

Determine whether each equation is a linear equation in two variables.

a. $x - 1.5y = -1.6$  
   b. $y = -2x$  
   c. $x + y^2 = 9$  
   d. $x = 5$

**Solution**

a. This is a linear equation in two variables because it is written in the form $Ax + By = C$ with $A = 1$, $B = -1.5$, and $C = -1.6$.
   
b. This is a linear equation in two variables because it can be written in the form $Ax + By = C$.
   
   $y = -2x$
   
   $2x + y = 0$  
   
   Add $2x$ to both sides.
   
   c. This is not a linear equation in two variables because $y$ is squared.
   
   d. This is a linear equation in two variables because it can be written in the form $Ax + By = C$.
   
   $x = 5$
   
   $x + 0y = 5$  
   
   Add $0 \cdot y$.

**Practice 1**

Determine whether each equation is a linear equation in two variables.

a. $3x + 2.7y = -5.3$  
   b. $x^2 + y = 8$  
   c. $y = 12$  
   d. $5x = -3y$

**Objective 2**

**Graphing linear equations by plotting ordered pair solutions.** From geometry, we know that a straight line is determined by just two points. Graphing a linear equation in two variables, then, requires that we find just two of its infinitely many solutions. Once we do so, we plot the solution points and draw the line connecting the points. Usually, we find a third solution as well, as a check.

**Example 2**

Graph the linear equation $2x + y = 5$.

**Solution**

Find three ordered pair solutions of $2x + y = 5$. To do this, choose a value for one variable, $x$ or $y$, and solve for the other variable. For example, let $x = 1$. Then $2x + y = 5$ becomes

$2(1) + y = 5$  

Replace $x$ with 1.

$2 + y = 5$  

Multiply.

$y = 3$  

Subtract 2 from both sides.

Since $y = 3$ when $x = 1$, the ordered pair $(1, 3)$ is a solution of $2x + y = 5$. Next, let $x = 0$.

$2x + y = 5$

$2(0) + y = 5$  

Replace $x$ with 0.

$0 + y = 5$

$y = 5$

The ordered pair $(0, 5)$ is a second solution.
The two solutions found so far allow us to draw the straight line that is the graph of all solutions of \(2x + y = 5\). However, we find a third ordered pair as a check. Let \(y = -1\).

\[
\begin{align*}
2x + y &= 5 \\
2x + (-1) &= 5 & \text{Replace } y \text{ with } -1. \\
2x &= 6 & \text{Add } 1 \text{ to both sides.} \\
x &= 3 & \text{Divide both sides by } 2. 
\end{align*}
\]

The third solution is \((3, -1)\). These three ordered pair solutions are listed in table form as shown. The graph of \(2x + y = 5\) is the line through the three points.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>

\[\text{Helpful Hint} \] All three points should fall on the same straight line. If not, check your ordered pair solutions for a mistake.

**Practice**

2. Graph the linear equation \(x + 3y = 9\).

**Example 3**

Graph the linear equation \(-5x + 3y = 15\).

**Solution**

Find three ordered pair solutions of \(-5x + 3y = 15\).

- **Let \(x = 0\).**
  \[
  \begin{align*}
  -5x + 3y &= 15 \\
  -5\cdot 0 + 3y &= 15 \\
  3y &= 15 \\
  y &= 5
  \end{align*}
  \]

- **Let \(y = 0\).**
  \[
  \begin{align*}
  -5x + 3y &= 15 \\
  -5x + 3\cdot 0 &= 15 \\
  -5x &= 15 \\
  x &= -3
  \end{align*}
  \]

- **Let \(x = -2\).**
  \[
  \begin{align*}
  -5x + 3y &= 15 \\
  -5(-2) + 3y &= 15 \\
  10 + 3y &= 15 \\
  y &= \frac{5}{3}
  \end{align*}
  \]

The ordered pairs are \((0, 5), (-3, 0), \) and \((-2, \frac{5}{3})\). The graph of \(-5x + 3y = 15\) is the line through the three points.

**Practice**

3. Graph the linear equation \(3x - 4y = 12\).
**EXAMPLE 4** Graph the linear equation $y = 3x$.

**Solution** To graph this linear equation, we find three ordered pair solutions. Since this equation is solved for $y$, choose three $x$ values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

Next, graph the ordered pair solutions listed in the table above and draw a line through the plotted points as shown on the next page. The line is the graph of $y = 3x$. Every point on the graph represents an ordered pair solution of the equation and every ordered pair solution is a point on this line.

**EXAMPLE 5** Graph the linear equation $y = -2x$.

**Solution** Find three ordered pair solutions, graph the solutions, and draw a line through the plotted solutions. To avoid fractions, choose $x$ values that are multiples of 3 to substitute in the equation. When a multiple of 3 is multiplied by $-\frac{1}{3}$, the result is an integer. See the calculations shown above the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$3$</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

**PRACTICE 4** Graph the linear equation $y = \frac{1}{2}x + 3$. 
Let’s compare the graphs in Examples 4 and 5. The graph of \( y = 3x \) tilts upward (as we follow the line from left to right) and the graph of \( y = -\frac{1}{3}x + 2 \) tilts downward (as we follow the line from left to right). We will learn more about the tilt, or slope, of a line in Section 3.4.

**Example 6** Graph the linear equation \( y = 3x + 6 \) and compare this graph with the graph of \( y = 3x \) in Example 4.

**Solution** Find ordered pair solutions, graph the solutions, and draw a line through the plotted solutions. We choose \( x \) values and substitute in the equation \( y = 3x + 6 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

The most startling similarity is that both graphs appear to have the same upward tilt as we move from left to right. Also, the graph of \( y = 3x \) crosses the \( y \)-axis at the origin, while the graph of \( y = 3x + 6 \) crosses the \( y \)-axis at 6. In fact, the graph of \( y = 3x + 6 \) is the same as the graph of \( y = 3x \) moved vertically upward 6 units.

**Practice 6** Graph the linear equation \( y = -2x + 3 \) and compare this graph with the graph of \( y = -2x \) in Practice 4.

Notice that the graph of \( y = 3x + 6 \) crosses the \( y \)-axis at 6. This happens because when \( x = 0 \), \( y = 3x + 6 \) becomes \( y = 3 \cdot 0 + 6 = 6 \). The graph contains the point \((0, 6)\), which is on the \( y \)-axis.

In general, if a linear equation in two variables is solved for \( y \), we say that it is written in the form \( y = mx + b \). The graph of this equation contains the point \((0, b)\) because when \( x = 0 \), \( y = mx + b \) is \( y = m \cdot 0 + b = b \).

The graph of \( y = mx + b \) crosses the \( y \)-axis at \((0, b)\).

We will review this again in Section 3.5.

Linear equations are often used to model real data as seen in the next example.
### Example 7  Estimating the Number of Medical Assistants

One of the occupations expected to have the most growth in the next few years is medical assistant. The number of people $y$ (in thousands) employed as medical assistants in the United States can be estimated by the linear equation $y = 31.8x + 180$, where $x$ is the number of years after the year 1995. (Source: Based on data from the Bureau of Labor Statistics)

**a.** Graph the equation.

**b.** Use the graph to predict the number of medical assistants in the year 2010.

#### Solution

**a.** To graph $y = 31.8x + 180$, choose $x$-values and substitute in the equation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>243.6</td>
</tr>
<tr>
<td>7</td>
<td>402.6</td>
</tr>
</tbody>
</table>

**b.** To use the graph to predict the number of medical assistants in the year 2010, we need to find the $y$-coordinate that corresponds to $x = 15$. (15 years after 1995 is the year 2010.) To do so, find 15 on the $x$-axis. Move vertically upward to the graphed line and then horizontally to the left. We approximate the number on the $y$-axis to be 655. Thus in the year 2010, we predict that there will be 655 thousand medical assistants. (The actual value, using 15 for $x$, is 657.)

### Practice 7

One of the occupations expected to have the most growth in the next few years is computer software application engineers. The number of people $y$ (in thousands) employed as computer software application engineers in the United States can be estimated by the linear equation $y = 22.2x + 371$, where $x$ is the number of years after 2000. (Source: Based on data from the Bureau of Labor Statistics)

**a.** Graph the equation.

**b.** Use the graph to predict the number of computer software application engineers in the year 2015.

#### Helpful Hint

Make sure you understand that models are mathematical approximations of the data for the known years. (For example, see the model in Example 7.) Any number of unknown factors can affect future years, so be cautious when using models to predict.
In this section, we begin an optional study of graphing calculators and graphing software packages for computers. These graphers use the same point plotting technique that was introduced in this section. The advantage of this graphing technology is, of course, that graphing calculators and computers can find and plot ordered pair solutions much faster than we can. Note, however, that the features described in these boxes may not be available on all graphing calculators.

The rectangular screen where a portion of the rectangular coordinate system is displayed is called a window. We call it a standard window for graphing when both the \(x\)- and \(y\)-axes show coordinates between \(-10\) and \(10\). This information is often displayed in the window menu on a graphing calculator as:

- \(\text{Xmin} = -10\)
- \(\text{Xmax} = 10\)
- \(\text{Xscl} = 1\) \(\text{Ymin} = -10\)
- \(\text{Ymax} = 10\)
- \(\text{Yscl} = 1\)

The scale on the \(x\)-axis is one unit per tick mark.
The scale on the \(y\)-axis is one unit per tick mark.

To use a graphing calculator to graph the equation \(y = 2x + 3\), press the \(Y=\) key and enter the keystrokes \(2x + 3\). The top row should now read \(Y_1 = 2x + 3\). Next press the \(\text{GRAPH}\) key, and the display should look like this:

![Graph of line with equation \(y = 2x + 3\)](image)

Use a standard window and graph the following linear equations. (Unless otherwise stated, use a standard window when graphing.)

1. \(y = -3x + 7\)
2. \(y = -x + 5\)
3. \(y = 2.5x - 7.9\)
4. \(y = -1.3x + 5.2\)
5. \(y = -\frac{3}{10}x + \frac{32}{5}\)
6. \(y = \frac{2}{9}x - \frac{22}{3}\)

### 3.2 EXERCISE SET

Determine whether each equation is a linear equation in two variables. See Example 1.

1. \(-x = 3y + 10\)
2. \(y = x - 15\)
3. \(x = y\)
4. \(x = y^3\)
5. \(x^2 + 2y = 0\)
6. \(0.01x - 0.2y = 8.8\)
7. \(y = -1\)
8. \(x = 25\)

For each equation, find three ordered pair solutions by completing the table. Then use the ordered pairs to graph the equation. See Examples 2 through 6.

9. \(x - y = 6\)
   | \(x\) | | \(y\) |
---|---|---|
   0 | | 0 |
   4 | | 2 |
   -1 | | -1 |

10. \(x - y = 4\)
    | \(x\) | | \(y\) |
---|---|---|
   0 | | 0 |
   4 | | 2 |
   -1 | | -1 |
11. \( y = -4x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
</tbody>
</table>

12. \( y = -5x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
</tbody>
</table>

13. \( y = \frac{1}{3}x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>-3</td>
<td>-2</td>
</tr>
</tbody>
</table>

14. \( y = \frac{1}{2}x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>-3</td>
<td>-1</td>
</tr>
</tbody>
</table>

15. \( y = -4x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

16. \( y = -5x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

MIXED PRACTICE
Graph each linear equation. See Examples 2 through 6.

17. \( x + y = 1 \)

18. \( x + y = 7 \)

19. \( x - y = -2 \)

20. \( -x + y = 6 \)

21. \( x - 2y = 6 \)

22. \( -x + 5y = 5 \)

23. \( y = 6x + 3 \)

24. \( y = -2x + 7 \)

25. \( x = -4 \)

26. \( y = 5 \)

27. \( y = 3 \)

28. \( x = -1 \)

29. \( y = x \)

30. \( y = -x \)

31. \( x = -3y \)

32. \( x = -5y \)

33. \( x + 3y = 9 \)

34. \( 2x + y = 2 \)

35. \( y = \frac{1}{2}x + 2 \)

36. \( y = \frac{1}{4}x + 3 \)

37. \( 3x - 2y = 12 \)

38. \( 2x - 7y = 14 \)

39. \( y = -3.5x + 4 \)

40. \( y = -1.5x - 3 \)

Graph each pair of linear equations on the same set of axes. Discuss how the graphs are similar and how they are different. See Example 6.

41. \( y = 5x; y = 5x + 4 \)

42. \( y = 2x; y = 2x + 5 \)

43. \( y = -2x; y = -2x - 3 \)

44. \( y = x; y = x - 7 \)

45. \( y = \frac{1}{2}x; y = \frac{1}{2}x + 2 \)

46. \( y = -\frac{1}{4}x; y = -\frac{1}{4}x + 3 \)

47. \( y = 5x + 5 \)

48. \( y = 5x - 4 \)

49. \( y = 5x - 1 \)

50. \( y = 5x + 2 \)

Solve. See Example 7.

51. Snowboarding is the fastest growing snow sport, and the number of participants has been increasing at a steady rate. The number of people involved in snowboarding (in millions) from the years 1997 to 2005 is given by the equation \( y = 0.5x + 3 \), where \( x \) is the number of years after 1997. (Source: Based on data from the National Sporting Goods Association.)
Section 3.2 Graphing Linear Equations

52. The revenue \( y \) (in billions of dollars) for Home Depot stores during the years 2000 through 2005 is given by the equation \( y = 7x + 45 \), where \( x \) is the number of years after 2000. (Source: Based on data from Home Depot stores)

a. Use this equation or a graph of it to complete the ordered pair \((8, \_\_\_\_\_)\).

b. Write a sentence explaining the meaning of the answer to part (a).

c. If this trend continues, how many snowboarders will there be in 2012?

53. The revenue \( y \) (in billions of dollars) for Home Depot stores during the years 2000 through 2005 is given by the equation \( y = 7x + 45 \), where \( x \) is the number of years after 2000. (Source: Based on data from Home Depot stores)

a. Use this equation or a graph of it to complete the ordered pair \((5, \_\_\_\_)\).

b. Write a sentence explaining the meaning of the answer to part (a).

c. If this trend continues, predict the revenue for Home Depot stores for the year 2015.

54. The revenue \( y \) (in billions of dollars) for Home Depot stores during the years 2000 through 2005 is given by the equation \( y = 7x + 45 \), where \( x \) is the number of years after 2000. (Source: Based on data from Home Depot stores)

a. Use this equation or a graph of it to complete the ordered pair \((5, \_\_\_\_)\).

b. Write a sentence explaining the meaning of the answer to part (a).

c. If this trend continues, predict the revenue for Home Depot stores for the year 2015.

55. The coordinates of three vertices of a rectangle are \((-2, 5), (4, 5), \) and \((-2, -1)\). Find the coordinates of the fourth vertex. See Section 3.1.

56. The coordinates of two vertices of a square are \((-3, -1)\) and \((2, -1)\). Find the coordinates of two pairs of points possible for the third and fourth vertices. See Section 3.1.

57. \(3(x - 2) + 5x = 6x - 16\)

58. \(5 + 7(x + 1) = 12 + 10x\)

59. \(3x + \frac{2}{5} = \frac{1}{10}\)

60. \(\frac{1}{6} + 2x = \frac{2}{3}\)

CONCEPT EXTENSIONS

Write each statement as an equation in two variables. Then graph the equation.

61. The \( y \)-value is 5 more than the \( x \)-value.

62. The \( y \)-value is twice the \( x \)-value.

63. Two times the \( x \)-value, added to three times the \( y \)-value is 6.

64. Five times the \( x \)-value, added to twice the \( y \)-value is \(-10\).

65. The perimeter of the trapezoid below is 22 centimeters. Write a linear equation in two variables for the perimeter. Find \( y \) if \( x \) is 3 cm.

66. The perimeter of the rectangle below is 50 miles. Write a linear equation in two variables for this perimeter. Use this equation to find \( x \) when \( y \) is 20.

67. Explain how to find ordered pair solutions of linear equations in two variables.

68. If \((a, b)\) is an ordered pair solution of \(x + y = 5\), is \((b, a)\) also a solution? Explain why or why not.

69. Graph the nonlinear equation \( y = x^2 \) by completing the table shown. Plot the ordered pairs and connect them with a smooth curve.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
</tbody>
</table>

70. Graph the nonlinear equation \( y = |x| \) by completing the table shown. Plot the ordered pairs and connect them. This curve is “V” shaped.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
</tbody>
</table>
3.3 INTERCEPTS

OBJECTIVES
1. Identify intercepts of a graph.
2. Graph a linear equation by finding and plotting intercepts.
3. Identify and graph vertical and horizontal lines.

OBJECTIVE 1 ◗ Identifying intercepts. In this section, we graph linear equations in two variables by identifying intercepts. For example, the graph of \( y = 4x - 8 \) is shown on right. Notice that this graph crosses the \( y \)-axis at the point \((0, -8)\). This point is called the \( y \)-intercept. Likewise, the graph crosses the \( x \)-axis at \((2, 0)\), and this point is called the \( x \)-intercept.

The intercepts are \((2, 0)\) and \((0, -8)\).

Helpful Hint
If a graph crosses the \( x \)-axis at \((-3, 0)\) and the \( y \)-axis at \((0, 7)\), then
\[
(-3, 0) \quad (0, 7)
\]
\(x\)-intercept \(y\)-intercept
Notice that for the \( y \)-intercept, the \( x \)-value is 0 and for the \( x \)-intercept, the \( y \)-value is 0.

Note: Sometimes in mathematics, you may see just the number 7 stated as the \( y \)-intercept, and stated as the \( x \)-intercept.

EXAMPLES Identify the \( x \)- and \( y \)-intercepts.

1. \( y = 4x - 8 \)

Solution
\( x\)-intercept: \((-3, 0)\)
\( y\)-intercept: \((0, 2)\)

Helpful Hint
Notice that any time \((0, 0)\) is a point of a graph, then it is an \( x \)-intercept and a \( y \)-intercept.

2. \( y = 4x - 8 \)

Solution
\( x\)-intercepts: \((-4, 0), (-1, 0)\)
\( y\)-intercept: \((0, 1)\)

3. \( y = 4x - 8 \)

Solution
\( x\)-intercept: \((0, 0)\)
\( y\)-intercept: \((0, 0)\)

4. \( y = 4x - 8 \)

Solution
\( x\)-intercept: \((2, 0)\)
\( y\)-intercept: none
Section 3.3 Intercepts

5.

\[ \text{Solution} \]

\( x \)-intercepts: \((-1, 0), (3, 0)\)

\( y \)-intercepts: \((0, 2), (0, -1)\)

**OBJECTIVE 2** Using intercepts to graph a linear equation. Given the equation of a line, intercepts are usually easy to find since one coordinate is 0.

One way to find the \( y \)-intercept of a line, given its equation, is to let \( x = 0 \), since a point on the \( y \)-axis has an \( x \)-coordinate of 0. To find the \( x \)-intercept of a line, let \( y = 0 \), since a point on the \( x \)-axis has a \( y \)-coordinate of 0.

**Finding \( x \)- and \( y \)-intercepts**

To find the \( x \)-intercept, let \( y = 0 \) and solve for \( x \).

To find the \( y \)-intercept, let \( x = 0 \) and solve for \( y \).
**EXAMPLE 6**  Graph $x - 3y = 6$ by finding and plotting intercepts.

**Solution**  Let $y = 0$ to find the $x$-intercept and let $x = 0$ to find the $y$-intercept.

\[
\begin{align*}
\text{Let } y &= 0 \\
\text{Let } x &= 0 \\
x - 3y &= 6 \\
x - 3(0) &= 6 \\
x - 0 &= 6 \\
-3y &= 6 \\
x &= 6 \\
y &= -2
\end{align*}
\]

The $x$-intercept is $(6, 0)$ and the $y$-intercept is $(0, -2)$. We find a third ordered pair solution to check our work. If we let $y = -1$, then $x = 3$. Plot the points $(6, 0)$, $(0, -2)$, and $(3, -1)$. The graph of $x - 3y = 6$ is the line drawn through these points, as shown.

**PRACTICE EXAMPLE 6**  Graph $x + 2y = -4$ by finding and plotting intercepts.

**EXAMPLE 7**  Graph $x = -2y$ by plotting intercepts.

**Solution**  Let $y = 0$ to find the $x$-intercept and $x = 0$ to find the $y$-intercept.

\[
\begin{align*}
\text{Let } y &= 0 \\
\text{Let } x &= 0 \\
x &= -2y \\
x &= -2(0) \\
x &= 0 \\
-2y &= 0 \\
x &= 0 \\
y &= 0
\end{align*}
\]

Both the $x$-intercept and $y$-intercept are $(0, 0)$. In other words, when $x = 0$, then $y = 0$, which gives the ordered pair $(0, 0)$. Also, when $y = 0$, then $x = 0$, which gives the same ordered pair $(0, 0)$. This happens when the graph passes through the origin. Since two points are needed to determine a line, we must find at least one more ordered pair that satisfies $x = -2y$. Let $y = -1$ to find a second ordered pair solution and let $y = 1$ as a checkpoint.

\[
\begin{align*}
\text{Let } y &= -1 \\
x &= -2(-1) \\
x &= 2
\end{align*}
\]

\[
\begin{align*}
\text{Let } y &= 1 \\
x &= -2(1) \\
x &= -2
\end{align*}
\]
The ordered pairs are (0, 0), (2, −1), and (−2, 1). Plot these points to graph $x = −2y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>−1</td>
</tr>
<tr>
<td>−2</td>
<td>1</td>
</tr>
</tbody>
</table>

**EXAMPLE 8** Graph $4x = 3y − 9$.

**Solution** Find the $x$- and $y$-intercepts, and then choose $x = 2$ to find a third checkpoint.

Let $y = 0$

$4x = 3(0) − 9$

$4x = −9$

Solve for $x$.

$x = −\frac{9}{4}$ or $x = −\frac{21}{4}$

Let $x = 0$

$4 \cdot 0 = 3y − 9$

$9 = 3y$

Solve for $y$.

$3 = y$

Let $x = 2$

$4(2) = 3y − 9$

$8 = 3y − 9$

Solve for $y$.

$17 = 3y$

$\frac{17}{3} = y$ or $y = \frac{5}{3}$

The ordered pairs are $\left(−2, \frac{1}{4}, 0\right)$, (0, 3), and $\left(2, \frac{5}{3}\right)$. The equation $4x = 3y − 9$ is graphed as follows.

**PRACTICE** Graph $3x = 2y + 4$.

**OBJECTIVE 3** Graphing vertical and horizontal lines. The equation $x = c$, where $c$ is a real number constant, is a linear equation in two variables because it can be written in the form $x + 0y = c$. The graph of this equation is a vertical line as shown in the next example.
EXAMPLE 9  Graph \( x = 2 \).

Solution  The equation \( x = 2 \) can be written as \( x + 0y = 2 \). For any \( y \)-value chosen, notice that \( x \) is 2. No other value for \( x \) satisfies \( x + 0y = 2 \). Any ordered pair whose \( x \)-coordinate is 2 is a solution of \( x + 0y = 2 \). We will use the ordered pair solutions \((2, 3)\), \((2, 0)\), and \((2, -3)\) to graph \( x = 2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
</tbody>
</table>

The graph is a vertical line with \( x \)-intercept \((2, 0)\). Note that this graph has no \( y \)-intercept because \( x \) is never 0.

PRACTICE 9  Graph \( y = 2 \).

Vertical Lines  The graph of \( x = c \), where \( c \) is a real number, is a vertical line with \( x \)-intercept \((c, 0)\).

EXAMPLE 10  Graph \( y = -3 \).

Solution  The equation \( y = -3 \) can be written as \( 0x + y = -3 \). For any \( x \)-value chosen, \( y \) is -3. If we choose 4, 1, and -2 as \( x \)-values, the ordered pair solutions are \((4, -3)\), \((1, -3)\), and \((-2, -3)\). Use these ordered pairs to graph \( y = -3 \). The graph is a horizontal line with \( y \)-intercept \((0, -3)\) and no \( x \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
</tbody>
</table>

PRACTICE 10  Graph \( x = -2 \).
Section 3.3 Intercepts

Horizontal Lines

The graph of $y = c$, where $c$ is a real number, is a horizontal line with $y$-intercept $(0, c)$.

Graphing Calculator Explorations

You may have noticed that to use the $\text{Y}=\text{key on a grapher to graph an equation, the equation must be solved for } y$. For example, to graph $2x + 3y = 7$, we solve this equation for $y$.

\[
2x + 3y = 7 \\
3y = -2x + 7 \\
\frac{3y}{3} = \frac{-2x + 7}{3} \\
y = -\frac{2}{3}x + \frac{7}{3}
\]

Simplify.

To graph $2x + 3y = 7$ or $y = -\frac{2}{3}x + \frac{7}{3}$ press the $\text{Y}=\text{key and enter}

\[
Y_1 = \frac{2}{3}x + \frac{7}{3}
\]

Graph each linear equation.

1. $x = 3.78y$  
2. $-2.61y = x$  
3. $3x + 7y = 21$

4. $-4x + 6y = 21$  
5. $-2.2x + 6.8y = 15.5$  
6. $5.9x - 0.8y = -10.4$
## VOCABULARY & READINESS CHECK

Use the choices below to fill in each blank. Some choices may be used more than once. Exercises 1 and 2 come from Section 3.2.

<table>
<thead>
<tr>
<th>x</th>
<th>vertical</th>
<th>x-intercept</th>
<th>linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>horizontal</td>
<td>y-intercept</td>
<td>standard</td>
</tr>
</tbody>
</table>

1. An equation that can be written in the form $Ax + By = C$ is called a _______ equation in two variables.
2. The form $Ax + By = C$ is called _______ form.
3. The graph of the equation $y = -1$ is a _______ line.
4. The graph of the equation $x = 5$ is a _______ line.
5. A point where a graph crosses the $y$-axis is called a(n) _______.
6. A point where a graph crosses the $x$-axis is called a(n) _______.
7. Given an equation of a line, to find the $x$-intercept (if there is one), let $____ = 0$ and solve for $____$.
8. Given an equation of a line, to find the $y$-intercept (if there is one), let $____ = 0$ and solve for $____$.

**Answer the following true or false.**

9. All lines have an $x$-intercept and a $y$-intercept.
10. The graph of $y = 4x$ contains the point $(0, 0)$.
11. The graph of $x + y = 5$ has an $x$-intercept of $(5, 0)$ and a $y$-intercept of $(0, 5)$.
12. The graph of $y = 5x$ contains the point $(5, 1)$.

### 3.3 EXERCISE SET

Identify the intercepts. See Examples 1 through 5.

1. [Graph 1](#)
2. [Graph 2](#)
3. [Graph 3](#)
4. [Graph 4](#)
5. [Graph 5](#)
6. [Graph 6](#)
Solve. See Example 1.

9. What is the greatest number of intercepts for a line?
10. What is the least number of intercepts for a line?
11. What is the least number of intercepts for a circle?
12. What is the greatest number of intercepts for a circle?

Graph each linear equation by finding and plotting its intercepts. See Examples 6 through 8.

13. $x - y = 3$
14. $x - y = -4$
15. $x = 5y$
16. $x = 2y$
17. $-x + 2y = 6$
18. $x - 2y = -8$
19. $2x - 4y = 8$
20. $2x + 3y = 6$
21. $y = 2x$
22. $y = -2x$
23. $y = 3x + 6$
24. $y = 2x + 10$

Graph each linear equation. See Examples 9 and 10.

25. $x = -1$
26. $y = 5$
27. $y = 0$
28. $x = 0$
29. $y = 7 = 0$
30. $x = -2 = 0$
31. $x + 3 = 0$
32. $y - 6 = 0$

**MIXED PRACTICE**

Graph each linear equation. See Examples 6 through 10.

33. $x = y$
34. $x = -y$
35. $x + 8y = 8$
36. $x + 3y = 9$
37. $5 = 6x - y$
38. $4 = x - 3y$
39. $-x + 10y = 11$
40. $-x + 9y = 10$
41. $x = -\frac{4}{2}$
42. $x = -\frac{3}{4}$
43. $y = \frac{1}{4}$
44. $y = \frac{2}{2}$
45. $y = -\frac{2}{3}x + 1$
46. $y = -\frac{3}{5}x + 3$
47. $4x - 6y + 2 = 0$
48. $9x - 6y + 3 = 0$

For Exercises 49 through 54, match each equation with its graph.

**E.**

- Graph E

**F.**

- Graph F

**49.** $y = 3$
**50.** $y = 2x + 2$
**51.** $x = -1$
**52.** $x = 3$
**53.** $y = 2x + 3$
**54.** $y = -2x$

**REVIEW AND PREVIEW**

Simplify. See Sections 1.5, 1.6, and 1.7.

55. $\frac{-6 - 3}{2 - 8}$
56. $\frac{-8 - (-2)}{-1 - 0}$
57. $\frac{-3 - (-2)}{0 - 6}$
58. $\frac{12 - 3}{10 - 9}$
59. $\frac{5}{3} - \frac{0}{5}$
60. $\frac{2 - 2}{3 - 5}$

**CONCEPT EXTENSIONS**

61. The revenue for the Disney Parks and Resorts $y$ (in millions) for the years 2003–2006 can be approximated by the equation $y = 181x + 6505$, where $x$ represents the number of years after 2003. (Source: Based on data from The Walt Disney Company)

- a. Find the $y$-intercept of this equation.
- b. What does the $y$-intercept mean?

62. The average price of a digital camera $y$ (in dollars) can be modeled by the linear equation $y = -78.1x + 491.8$ where $x$ represents the number of years after 2000. (Source: NPD Techworld)

- a. Find the $y$-intercept of this equation.
- b. What does this $y$-intercept mean?

63. Since 2002, admissions at movie theaters have been in a decline. The number of people $y$ (in billions) who go to movie theaters each year can be estimated by the equation $y = -0.075x + 1.65$, where $x$ represents the number of years...
since 2002. *(Source: Based on data from Motion Picture Association of America)*

a. Find the \(x\)-intercept of this equation.

b. What does this \(x\)-intercept mean?

c. Use part (b) to comment on the limitations of using equations to model real data.

**64.** The price of admission to a movie theater has been steadily increasing. The price of regular admission \(y\) (in dollars) to a movie theater may be represented by the equation \(y = 0.2x + 5.42\), where \(x\) is the number of years after 2000. *(Source: Based on data from Motion Picture Association of America)*

a. Find the \(x\)-intercept of this equation.

b. What does this \(x\)-intercept mean?

c. Use part (b) to comment on the limitations of using equations to model real data.

**65.** The production supervisor at Alexandra’s Office Products finds that it takes 3 hours to manufacture a particular office chair and 6 hours to manufacture a computer desk. A total of 1200 hours is available to produce office chairs and desks of this style. The linear equation that models this situation is \(3x + 6y = 1200\), where \(x\) represents the number of chairs produced and \(y\) the number of desks manufactured.

\(\textbf{\Delta a.}\) Complete the ordered pair solution \((0, \ )\) of this equation. Describe the manufacturing situation that corresponds to this solution.

\(\textbf{\Delta b.}\) Complete the ordered pair solution \(( , 0)\) of this equation. Describe the manufacturing situation that corresponds to this solution.

\(\textbf{\Delta c.}\) Use the ordered pairs found above and graph the equation \(3x + 6y = 1200\).

\(\textbf{\Delta d.}\) If 50 computer desks are manufactured, find the greatest number of chairs that they can make.

**66.** Two lines in the same plane that do not intersect are called \textbf{parallel} lines.

**67.** Draw a line parallel to the line \(x = 5\) that intersects the \(x\)-axis at \((1, 0)\). What is the equation of this line?

**68.** Draw a line parallel to the line \(y = -1\) that intersects the \(y\)-axis at \((0, -4)\). What is the equation of this line?

**69.** Discuss whether a vertical line ever has a \(y\)-intercept.

**70.** Explain why it is a good idea to use three points to graph a linear equation.

**71.** Explain how to find intercepts.

---

**STUDY SKILLS BUILDER**

**Are You Familiar with Your Textbook Supplements?**

Below is a review of some of the student supplements available for additional study. Check to see if you are using the ones most helpful to you.

- **Chapter Test Prep Videos on CD.** This CD is found with your textbook and contains video clip solutions to the Chapter Test exercises in this text. You will find this extremely useful when studying for chapter tests.
- **Lecture Videos on CD-ROM.** These are keyed to each section of the text. The material is presented by me, Elayn Martin-Gay, and I have placed a \(\blacklozenge\) by the exercises in the text that I have worked on the video.
- **The Student Solutions Manual.** This contains worked out solutions to odd-numbered exercises as well as every exercise in the Integrated Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews.
- **Pearson Tutor Center.** Mathematics questions may be phoned, faxed, or emailed to this center.
- **MyMathLab** is a text-specific online course. MathXL is an online homework, tutorial, and assessment system. Take a moment and determine whether these are available to you.

As usual, your instructor is your best source of information.

**Let’s see how you are doing with textbook supplements.**

1. Name one way the Lecture Videos can be helpful to you.
2. Name one way the Chapter Test Prep Video can help you prepare for a chapter test.
3. List any textbook supplements that you have found useful.
4. Have you located and visited a learning resource lab located on your campus?
5. List the textbook supplements that are currently housed in your campus’ learning resource lab.
OBJECTIVES
1. Find the slope of a line given two points of the line.
2. Find the slope of a line given its equation.
3. Find the slopes of horizontal and vertical lines.
4. Compare the slopes of parallel and perpendicular lines.
5. Slope as a rate of change.

OBJECTIVE 1 Finding the slope of a line given two points of the line. Thus far, much of this chapter has been devoted to graphing lines. You have probably noticed by now that a key feature of a line is its slant or steepness. In mathematics, the slant or steepness of a line is formally known as its slope. We measure the slope of a line by the ratio of vertical change to the corresponding horizontal change as we move along the line.

On the line below, for example, suppose that we begin at the point (1, 2) and move to the point (4, 6). The vertical change is the change in \( y \)-coordinates: \( 6 - 2 = 4 \) units. The corresponding horizontal change is the change in \( x \)-coordinates: \( 4 - 1 = 3 \) units. The ratio of these changes is

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{4}{3}
\]

The slope of this line, then, is \( \frac{4}{3} \). This means that for every 4 units of change in \( y \)-coordinates, there is a corresponding change of 3 units in \( x \)-coordinates.

Helpful Hint
It makes no difference what two points of a line are chosen to find its slope. The slope of a line is the same everywhere on the line.

To find the slope of a line, then, choose two points of the line. Label the two \( x \)-coordinates of two points, \( x_1 \) and \( x_2 \) (read “\( x \) sub one” and “\( x \) sub two”), and label the corresponding \( y \)-coordinates \( y_1 \) and \( y_2 \).

The vertical change or rise between these points is the difference in the \( y \)-coordinates: \( y_2 - y_1 \). The horizontal change or run between the points is the
difference of the $x$-coordinates: $x_2 - x_1$. The slope of the line is the ratio of $y_2 - y_1$ to $x_2 - x_1$, and we traditionally use the letter $m$ to denote slope $m = \frac{y_2 - y_1}{x_2 - x_1}$.

**Slope of a Line**

The slope $m$ of the line containing the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{as long as } x_2 \neq x_1$$

**EXAMPLE 1** Find the slope of the line through $(-1, 5)$ and $(2, -3)$. Graph the line.

**Solution** If we let $(x_1, y_1)$ be $(-1, 5)$, then $x_1 = -1$ and $y_1 = 5$. Also, let $(x_2, y_2)$ be $(2, -3)$ so that $x_2 = 2$ and $y_2 = -3$. Then, by the definition of slope,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{2 - (-1)} = \frac{-8}{3} = -\frac{8}{3}$$

The slope of the line is $-\frac{8}{3}$.

**PRACTICE** Find the slope of the line through $(-4, 11)$ and $(2, 5)$.

**Helpful Hint**

When finding slope, it makes no difference which point is identified as $(x_1, y_1)$ and which is identified as $(x_2, y_2)$. Just remember that whatever $y$-value is first in the numerator, its corresponding $x$-value is first in the denominator. Another way to calculate the slope in Example 1 is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-3)}{-1 - 2} = \frac{8}{-3} = -\frac{8}{3} \quad \text{or} \quad \frac{8}{3} \quad \text{Same slope as found in Example 1.}$$

**Answer to Concept Check:**

$$m = \frac{3}{2}$$

**Concept Check**

The points $(-2, -5), (0, -2), (4, 4)$, and $(10, 13)$ all lie on the same line. Work with a partner and verify that the slope is the same no matter which points are used to find slope.
Example 2  Find the slope of the line through \((-1, -2)\) and \((2, 4)\). Graph the line.

Solution  Let \((x_1, y_1)\) be \((2, 4)\) and \((x_2, y_2)\) be \((-1, -2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{-1 - 2} = \frac{-6}{-3} = 2
\]

Helpful Hint

The slope for Example 2 is the same if we let \((x_1, y_1)\) be \((-1, -2)\) and \((x_2, y_2)\) be \((2, 4)\).

Concept Check

What is wrong with the following slope calculation for the points \((3, 5)\) and \((-2, 6)\)?

\[
m = \frac{5 - 6}{-2 - 3} = \frac{-1}{-5} = \frac{1}{5}
\]

Notice that the slope of the line in Example 1 is negative, whereas the slope of the line in Example 2 is positive. Let your eye follow the line with negative slope from left to right and notice that the line “goes down.” Following the line with positive slope from left to right, notice that the line “goes up.” This is true in general.

Objective 2  Finding the slope of a line given its equation.  As we have seen, the slope of a line is defined by two points on the line. Thus, if we know the equation of a line, we can find its slope by finding two of its points. For example, let’s find the slope of the line

\[y = 3x + 2\]

To find two points, we can choose two values for \(x\) and substitute to find corresponding \(y\)-values. If \(x = 0\), for example, \(y = 3 \cdot 0 + 2\) or \(y = 2\). If \(x = 1\), \(y = 3 \cdot 1 + 2\) or \(y = 5\). This gives the ordered pairs \((0, 2)\) and \((1, 5)\). Using the definition for slope, we have

\[
m = \frac{5 - 2}{1 - 0} = \frac{3}{1} = 3\]

The slope is 3.

Notice that the slope, 3, is the same as the coefficient of \(x\) in the equation \(y = 3x + 2\).

Also, recall from Section 3.2 that the graph of an equation of the form \(y = mx + b\) has \(y\)-intercept \((0, b)\).

This means that the \(y\)-intercept of the graph of \(y = 3x + 2\) is \((0, 2)\). This is true in general.
When a linear equation is written in the form \( y = mx + b \), not only is \((0, b)\) the \(y\)-intercept of the line, but \(m\) is its slope. The form \( y = mx + b \) is appropriately called the **slope-intercept form**.

\[ y = mx + b \]

**Slope-Intercept Form**

When a linear equation in two variables is written in slope-intercept form,

\[ y = mx + b \]

\(m\) is the slope of the line and \((0, b)\) is the \(y\)-intercept of the line.

**EXAMPLE 3** Find the slope and \(y\)-intercept of the line whose equation is \( y = \frac{3}{4}x + 6 \).

**Solution** The equation is in slope-intercept form, \( y = mx + b \).

\[ y = \frac{3}{4}x + 6 \]

The coefficient of \(x\), \(\frac{3}{4}\), is the slope and the constant term, \(6\), is the \(y\)-value of the \(y\)-intercept, \((0, 6)\).

**PRACTICE 3** Find the slope and \(y\)-intercept of the line whose equation is \( y = \frac{2}{3}x - 2 \).

**EXAMPLE 4** Find the slope and the \(y\)-intercept of the line whose equation is \( 5x + y = 2 \).

**Solution** Write the equation in slope-intercept form by solving the equation for \(y\).

\[ 5x + y = 2 \]

\[ y = -5x + 2 \quad \text{Subtract } 5x \text{ from both sides.} \]

The coefficient of \(x\), \(-5\), is the slope and the constant term, \(2\), is the \(y\)-value of the \(y\)-intercept, \((0, 2)\).

**PRACTICE 4** Find the slope and \(y\)-intercept of the line whose equation is \( 6x - y = 5 \).

**EXAMPLE 5** Find the slope and the \(y\)-intercept of the line whose equation is \( 3x - 4y = 4 \).

**Solution** Write the equation in slope-intercept form by solving for \(y\).

\[ 3x - 4y = 4 \]

\[ -4y = -3x + 4 \quad \text{Subtract } 3x \text{ from both sides.} \]

\[ \frac{-4y}{-4} = \frac{-3x}{-4} + \frac{4}{-4} \quad \text{Divide both sides by } -4. \]

\[ y = \frac{3}{4}x - 1 \quad \text{Simplify.} \]

The coefficient of \(x\), \(\frac{3}{4}\), is the slope, and the \(y\)-intercept is \((0, -1)\).

**PRACTICE 5** Find the slope and the \(y\)-intercept of the line whose equation is \( 5x + 2y = 8 \). 

OBJECTIVE 3 Finding slopes of horizontal and vertical lines. Recall that if a line tilts upward from left to right, its slope is positive. If a line tilts downward from left to right, its slope is negative. Let's now find the slopes of two special lines, horizontal and vertical lines.

EXAMPLE 6 Find the slope of the line $y = -1$.

Solution Recall that $y = -1$ is a horizontal line with $y$-intercept $(0, -1)$. To find the slope, find two ordered pair solutions of $y = -1$. Solutions of $y = -1$ must have a $y$-value of $-1$. Let's use points $(2, -1)$ and $(-3, -1)$, which are on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-1)}{-3 - 2} = \frac{0}{-5} = 0$$

The slope of the line $y = -1$ is 0 and its graph is shown.

PRACTICE 6 Find the slope of the line $y = 3$.

Any two points of a horizontal line will have the same $y$-values. This means that the $y$-values will always have a difference of 0 for all horizontal lines. Thus, all horizontal lines have a slope 0.

EXAMPLE 7 Find the slope of the line $x = 5$.

Solution Recall that the graph of $x = 5$ is a vertical line with $x$-intercept $(5, 0)$.

To find the slope, find two ordered pair solutions of $x = 5$. Solutions of $x = 5$ must have an $x$-value of 5. Let's use points $(3, 0)$ and $(5, 4)$, which are on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{5 - 3} = \frac{4}{0}$$

Since $\frac{4}{0}$ is undefined, we say the slope of the vertical line $x = 5$ is undefined, and its graph is shown.

PRACTICE 7 Find the slope of the line $x = -4$.

Any two points of a vertical line will have the same $x$-values. This means that the $x$-values will always have a difference of 0 for all vertical lines. Thus all vertical lines have undefined slope.

Helpful Hint Slope of 0 and undefined slope are not the same. Vertical lines have undefined slope or no slope, while horizontal lines have a slope of 0.
Here is a general review of slope.

**Summary of Slope**
Slope \( m \) of the line through \((x_1, y_1)\) and \((x_2, y_2)\) is given by the equation
\[
m = \frac{y_2 - y_1}{x_2 - x_1}.
\]

**OBJECTIVE 4 ▶ Slopes of parallel and perpendicular lines.** Two lines in the same plane are parallel if they do not intersect. Slopes of lines can help us determine whether lines are parallel. Parallel lines have the same steepness, so it follows that they have the same slope.

For example, the graphs of
\[
y = -2x + 4
\]
and
\[
y = -2x - 3
\]
are shown. These lines have the same slope, \(-2\). They also have different \(y\)-intercepts, so the lines are parallel. (If the \(y\)-intercepts were the same also, the lines would be the same.)

**Parallel Lines**
Nonvertical parallel lines have the same slope and different \(y\)-intercepts.

Two lines are perpendicular if they lie in the same plane and meet at a 90° (right) angle. How do the slopes of perpendicular lines compare? The product of the slopes of two perpendicular lines is \(-1\).
For example, the graphs of 

\[ y = 4x + 1 \]

and

\[ y = -\frac{1}{4}x - 3 \]

are shown. The slopes of the lines are 4 and \(-\frac{1}{4}\). Their product is \(-1\), so the lines are perpendicular.

### Perpendicular Lines

If the product of the slopes of two lines is \(-1\), then the lines are perpendicular.

(Two nonvertical lines are perpendicular if the slopes of one is the negative reciprocal of the slope of the other.)

---

**Helpful Hint**

Here are examples of numbers that are negative (opposite) reciprocals.

<table>
<thead>
<tr>
<th>Number</th>
<th>Negative Reciprocal</th>
<th>Their Product Is (-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2}{3})</td>
<td>(-\frac{3}{2})</td>
<td>(\frac{2}{3} \cdot \frac{-3}{2} = -1)</td>
</tr>
<tr>
<td>(-\frac{5}{1})</td>
<td>(\frac{1}{5})</td>
<td>((-\frac{5}{1}) \cdot \frac{1}{5} = -1)</td>
</tr>
</tbody>
</table>

---

**Helpful Hint**

Here are a few important facts about vertical and horizontal lines.

- Two distinct vertical lines are parallel.
- Two distinct horizontal lines are parallel.
- A horizontal line and a vertical line are always perpendicular.

---

**Example 8**

Determine whether each pair of lines is parallel, perpendicular, or neither.

**a.** \(y = -\frac{1}{5}x + 1\)

\[2x + 10y = 3\]

**Solution**

a. The slope of the line \(y = -\frac{1}{5}x + 1\) is \(-\frac{1}{5}\). We find the slope of the second line by solving its equation for \(y\).

\[2x + 10y = 3\]

Subtract 2\(x\) from both sides.

\[10y = -2x + 3\]

Divide both sides by 10.

\[y = -\frac{2}{10}x + \frac{3}{10}\]

Simplify.

\[y = -\frac{1}{5}x + \frac{3}{10}\]

The slope of this line is \(-\frac{1}{5}\) also. Since the lines have the same slope and different \(y\)-intercepts, they are parallel, as shown in the figure on the next page.
b. To find each slope, we solve each equation for $y$.

\[
\begin{align*}
  x + y &= 3 & -x + y &= 4 \\
  y &= -x + 3 & y &= x + 4 \\
\end{align*}
\]

The slope is $-1$. The slope is 1.

The slopes are not the same, so the lines are not parallel. Next we check the product of the slopes: $(-1)(1) = -1$. Since the product is $-1$, the lines are perpendicular, as shown in the figure.

\[
\begin{align*}
  y &= -\frac{1}{3}x + 1 \\
  2x + 10y &= 3
\end{align*}
\]

\[
\begin{align*}
  -x + y &= 4 \\
  x + y &= 3
\end{align*}
\]

c. We solve each equation for $y$ to find each slope. The slopes are $-3$ and $-\frac{2}{3}$. The slopes are not the same and their product is not $-1$. Thus, the lines are neither parallel nor perpendicular.

**Practice**

Determine whether each pair of lines is parallel, perpendicular, or neither.

a. $y = -5x + 1$

\[
\begin{align*}
  x - 5y &= 10
\end{align*}
\]

b. $x + y = 11$

\[
\begin{align*}
  2x + y &= 11
\end{align*}
\]

c. $2x + 3y = 21$

\[
\begin{align*}
  6y &= -4x - 2
\end{align*}
\]

**Concept Check**

Consider the line $-6x + 2y = 1$.

a. Write the equations of two lines parallel to this line.

b. Write the equations of two lines perpendicular to this line.

**OBJECTIVE 5** Slope as a rate of change.** Slope can also be interpreted as a rate of change. In other words, slope tells us how fast $y$ is changing with respect to $x$. To see this, let’s look at a few of the many real-world applications of slope. For example, the pitch of a roof, used by builders and architects, is its slope. The pitch of the roof on the left is $\frac{7}{10}$ rise over run. This means that the roof rises vertically 7 feet for every horizontal 10 feet. The rate of change for the roof is 7 vertical feet ($y$) per 10 horizontal feet ($x$).

The grade of a road is its slope written as a percent. A 7% grade, as shown below, means that the road rises (or falls) 7 feet for every horizontal 100 feet. (Recall that $7\% = \frac{7}{100}$) Here, the slope of $\frac{7}{100}$ gives us the rate of change. The road rises (in our diagram) 7 vertical feet ($y$) for every 100 horizontal feet ($x$).
Section 3.4 Slope and Rate of Change

**EXAMPLE 9** Finding the Grade of a Road

At one part of the road to the summit of Pikes Peak, the road rises at a rate of 15 vertical feet for a horizontal distance of 250 feet. Find the grade of the road.

*Solution* Recall that the grade of a road is its slope written as a percent.

\[
\text{grade} = \frac{\text{rise}}{\text{run}} = \frac{15}{250} = 0.06 = 6\%
\]

The grade is 6%.

**PRACTICE**

9. One part of the Mt. Washington (New Hampshire) cog railway rises about 1794 feet over a horizontal distance of 7176 feet. Find the grade of this part of the railway.

---

**EXAMPLE 10** Finding the Slope of a Line

The following graph shows the cost \(y\) (in cents) of a nationwide long-distance telephone call from Texas with a certain telephone-calling plan, where \(x\) is the length of the call in minutes. Find the slope of the line and attach the proper units for the rate of change. Then write a sentence explaining the meaning of slope in this application.

*Solution* Use (2, 34) and (6, 62) to calculate slope.

\[
m = \frac{62 - 34}{6 - 2} = \frac{28}{4} = 7 \text{ cents per minute}
\]

This means that the rate of change of a phone call is 7 cents per 1 minute or the cost of the phone call is 7 cents per minute.

**PRACTICE**

10. The following graph shows the cost \(y\) (in dollars) of having laundry done at the Wash-n-Fold, where \(x\) is the number of pounds of laundry. Find the slope of the line, and attach the proper units for the rate of change.
Graphing Calculator Explorations

It is possible to use a grapher to sketch the graph of more than one equation on the same set of axes. This feature can be used to confirm our findings from Section 3.2 when we learned that the graph of an equation written in the form $y = mx + b$ has a $y$-intercept of $b$. For example, graph the equations $y = \frac{2}{5}x$, $y = \frac{2}{5}x + 7$, and $y = \frac{2}{5}x - 4$ on the same set of axes. To do so, press the $Y=$ key and enter the equations on the first three lines.

$Y_1 = \left( \frac{2}{5} \right)x$
$Y_2 = \left( \frac{2}{5} \right)x + 7$
$Y_3 = \left( \frac{2}{5} \right)x - 4$

The screen should look like:

Notice that all three graphs appear to have the same positive slope. The graph of $y = \frac{2}{5}x + 7$ is the graph of $y = \frac{2}{5}x$ moved 7 units upward with a $y$-intercept of 7. Also, the graph of $y = \frac{2}{5}x - 4$ is the graph of $y = \frac{2}{5}x$ moved 4 units downward with a $y$-intercept of −4.

Graph the equations on the same set of axes. Describe the similarities and differences in their graphs:

1. $y = 3.8x$, $y = 3.8x - 3$, $y = 3.8x + 6$
2. $y = -4.9x$, $y = -4.9x + 1$, $y = -4.9x + 8$
3. $y = \frac{1}{4}x$, $y = \frac{1}{4}x + 5$, $y = \frac{1}{4}x - 8$
4. $y = -\frac{3}{4}x$, $y = -\frac{3}{4}x - 5$, $y = -\frac{3}{4}x + 6$

VOCABULARY & READINESS CHECK

Use the choices below to fill in each blank. Not all choices will be used.

$m \quad x \quad 0 \quad positive \quad undefined$
$b \quad y \quad slope \quad negative$

1. The measure of the steepness or tilt of a line is called ________.
2. If an equation is written in the form $y = mx + b$, the value of the letter ________ is the value of the slope of the graph.
3. The slope of a horizontal line is ________.
4. The slope of a vertical line is ________.
5. If the graph of a line moves upward from left to right, the line has ________ slope.
6. If the graph of a line moves downward from left to right, the line has _______ slope.

7. Given two points of a line, slope = \frac{\text{change in } y}{\text{change in } x}.

State whether the slope of the line is positive, negative, 0, or is undefined.

8. \[ y = x \]

9. \[ y = x \]

10. \[ y = x \]

11. \[ y = x \]

Decide whether a line with the given slope is upward, downward, horizontal, or vertical.

12. \[ m = \frac{7}{6} \]

13. \[ m = -3 \]

14. \[ m = 0 \]

15. \[ m \] is undefined.

### EXERCISE SET

Find the slope of the line that passes through the given points. See Examples 1 and 2.

1. \((-1, 5)\) and \((6, -2)\)

2. \((3, 1)\) and \((2, 6)\)

3. \((-4, 3)\) and \((-4, 5)\)

4. \((6, -6)\) and \((6, 2)\)

5. \((-2, 8)\) and \((1, 6)\)

6. \((4, -3)\) and \((2, 2)\)

7. \((5, 1)\) and \((-2, 1)\)

8. \((0, 13)\) and \((-4, 13)\)

Find the slope of each line if it exists. See Examples 1 and 2.

9. \[ y = x \]

10. \[ y = x \]

11. \[ y = x \]

12. \[ y = x \]

For each graph, determine which line has the greater slope.

13. \[ \text{line 1} \]

14. \[ \text{line 1} \]

15. \[ \text{line 1} \]

16. \[ \text{line 1} \]

17. \[ \text{line 1} \]

18. \[ \text{line 1} \]
In Exercises 19 through 24, match each line with its slope.

A. \( m = 0 \)  
B. undefined slope  
C. \( m = 3 \)

D. \( m = -1/2 \)  
E. \( m = -3/4 \)

Find the slope of each line. See Examples 6 and 7.

25. \( x = 6 \)  
26. \( y = 4 \)  
27. \( y = -4 \)  
28. \( x = 2 \)  
29. \( x = -3 \)  
30. \( y = -11 \)  
31. \( y = 0 \)  
32. \( x = 0 \)

**MIXED PRACTICE**

Find the slope of each line. See Examples 3 through 7.

33. \( y = 5x - 2 \)  
34. \( y = -2x + 6 \)  
35. \( y = -0.3x + 2.5 \)  
36. \( y = -7.6x - 0.1 \)  
37. \( 2x + y = 7 \)  
38. \( -5x + y = 10 \)  
39. \( 2x - 3y = 10 \)  
40. \( 3x - 5y = 1 \)  
41. \( x = 1 \)  
42. \( y = -2 \)  
43. \( x = 2y \)  
44. \( x = -4y \)  
45. \( y = -3 \)  
46. \( x = 5 \)

47. \( -3x - 4y = 6 \)  
48. \( -4x - 7y = 9 \)  
49. \( 20x - 5y = 1.2 \)  
50. \( 24x - 3y = 5.7 \)

\( \triangle \) Determine whether each pair of lines is parallel, perpendicular, or neither. See Example 8.

51. \( y = \frac{2}{9}x + 3 \)  
52. \( y = \frac{1}{5}x + 20 \)

\( y = -\frac{2}{9}x \)  
\( y = -\frac{1}{5}x \)

53. \( x - 3y = -6 \)  
54. \( y = 4x - 2 \)  
55. \( 6x = 5y + 1 \)  
56. \( -x + 2y = -2 \)  
57. \( 6 + 4x = 3y \)  
58. \( 10 + 3x = 5y \)  
59. \( 3x + 4y = 8 \)  
60. \( 5x + 3y = 1 \)

**The pitch of a roof is its slope. Find the pitch of each roof shown. See Example 9.**

59.

60.

**The grade of a road is its slope written as a percent. Find the grade of each road shown. See Example 9.**

61.

62.

63. One of Japan’s superconducting “bullet” trains is researched and tested at the Yamanashi Maglev Test Line near Otsuki City. The steepest section of the track has a rise of 2580 meters for a horizontal distance of 6450 meters. What is the grade of this section of track? (Source: Japan Railways Central Co.)
64. Professional plumbers suggest that a sewer pipe should rise 0.25 inch for every horizontal foot. Find the recommended slope for a sewer pipe. Round to the nearest hundredth.

65. The steepest street is Baldwin Street in Dunedin, New Zealand. It has a maximum rise of 10 meters for a horizontal distance of 12.66 meters. Find the grade of this section of road. Round to the nearest whole percent. (Source: The Guinness Book of Records)

66. According to federal regulations, a wheelchair ramp should rise no more than 1 foot for a horizontal distance of 12 feet. Write the slope as a grade. Round to the nearest tenth of a percent.

67. This graph approximates the number of U.S. households that have personal computers $y$ (in millions) for year $x$.

68. This graph approximates the number of Attention Deficit Hyperactivity Disorder (ADHD) prescriptions for children under 18 for the year $x$. (Source: Centers for Disease Control and Prevention (CDC) National Center for Health Statistics)

69. The graph below shows the total cost $y$ (in dollars) of owning and operating a compact car where $x$ is the number of miles driven.

70. The graph below shows the total cost $y$ (in dollars) of owning and operating a standard pickup truck, where $x$ is the number of miles driven.

REVIEW AND PREVIEW
Solve each equation for $y$. See Section 2.5.

71. $y - (-6) = 2(x - 4)$
72. $y - 9 = -9(x - 6)$
73. $y - 4 = -6(x + 2)$
74. $y - (-3) = 4(x + 5)$

CONCEPT EXTENSIONS

△ FIND THE SLOPE OF THE LINE THAT IS (a) PARALLEL AND (b) PERPENDICULAR TO THE LINE THROUGH EACH PAIR OF POINTS.

75. $(-3, -3)$ and $(0, 0)$
76. $(6, -2)$ and $(1, 4)$
77. $(-8, -4)$ and $(3, 5)$
78. $(6, -1)$ and $(-4, -10)$

Solve. See a Concept Check in this section.

79. Verify that the points $(2, 1), (0, 0), (-2, -1)$ and $(-4, -2)$ are all on the same line by computing the slope between each pair of points. (See the first Concept Check.)
80. Given the points \((2, 3)\) and \((-5, 1)\), can the slope of the line through these points be calculated by \(\frac{1 - 3}{2 - (-5)}\)? Why or why not? (See the second Concept Check.)

81. Write the equations of three lines parallel to \(10x - 5y = -7\). (See the third Concept Check.)

82. Write the equations of two lines perpendicular to \(10x - 5y = -7\). (See the third Concept Check.)

The following line graph shows the average fuel economy (in miles per gallon) by mid-size passenger automobiles produced during each of the model years shown. Use this graph to answer Exercises 83 through 88.

83. What was the average fuel economy (in miles per gallon) for automobiles produced during 2001?

84. Find the decrease in average fuel economy for automobiles between the years 2003 to 2004.

85. During which of the model years shown was average fuel economy the lowest?

86. During which of the model years shown was average fuel economy the highest?

87. What line segment has the greatest slope?

88. What line segment has the least positive slope?

89. Find \(x\) so that the pitch of the roof is \(\frac{2}{3}\).

90. Find \(x\) so that the pitch of the roof is \(\frac{4}{5}\).

91. The average price of an acre of U.S. farmland was $1132 in 2001. In 2006, the price of an acre rose to approximately $1657. (Source: National Agricultural Statistics Service)
   a. Write two ordered pairs of the form (year, price of acre)
   b. Find the slope of the line through the two points.
   c. Write a sentence explaining the meaning of the slope as a rate of change.

92. There were approximately 14,774 kidney transplants performed in the United States in 2002. In 2006, the number of kidney transplants performed in the United States rose to 15,722. (Source: Organ Procurement and Transplantation Network)
   a. Write two ordered pairs of the form (year, number of kidney transplants).
   b. Find the slope of the line between the two points.
   c. Write a sentence explaining the meaning of the slope as a rate of change.

93. Show that a triangle with vertices at the points \((1, 1)\), \((-4, 4)\), and \((-3, 0)\) is a right triangle.

94. Show that the quadrilateral with vertices \((1, 3), (2, 1), (-4, 0),\) and \((-3, -2)\) is a parallelogram.

95. Find the slope of the line through the given points.
   (2.1, 6.7) and (-8.3, 9.3)
   (2.3, 0.2) and (7.9, 5.1)
   (14.3, -10.1) and (9.8, -2.9)

96. The graph of \(y = -\frac{1}{3}x + 2\) has a slope of \(-\frac{1}{3}\). The graph of \(y = -2x + 2\) has a slope of \(-2\). The graph of \(y = -4x + 2\) has a slope of \(-4\). Graph all three equations on a single coordinate system. As the absolute value of the slope becomes larger, how does the steepness of the line change?

99. The graph of \(y = -\frac{1}{3}x + 2\) has a slope of \(-\frac{1}{3}\). The graph of \(y = 3x\) has a slope of 3. The graph of \(y = 5x\) has a slope of 5. Graph all three equations on a single coordinate system. As slope becomes larger, how does the steepness of the line change?
INTEGRATED REVIEW SUMMARY ON SLOPE & GRAPHING LINEAR EQUATIONS

Sections 3.1–3.4

Find the slope of each line.

1. 
2. 
3. 
4. 

Graph each linear equation.

5. \( y = -2x \) 
6. \( x + y = 3 \) 
7. \( x = -1 \) 
8. \( y = 4 \) 
9. \( x - 2y = 6 \) 
10. \( y = 3x + 2 \) 
11. \( 5x + 3y = 15 \) 
12. \( 2x - 4y = 8 \)

Determine whether the lines through the points are parallel, perpendicular, or neither.

13. \( y = \frac{-1}{3}x + \frac{1}{3} \)

\( 3x = -15y \)

14. \( x - y = \frac{1}{2} \)

\( 3x - y = \frac{1}{2} \)

15. In the years 2002 through 2005 the number of admissions to movie theaters in the United States can be modeled by the linear equation \( y = -75x + 1650 \) where \( x \) is years after 2002 and \( y \) is admissions in millions. (Source: Motion Picture Assn. of America)

a. Find the \( y \)-intercept of this line.

b. Write a sentence explaining the meaning of this intercept.

c. Find the slope of this line.

d. Write a sentence explaining the meaning of the slope as a rate of change.

16. Online advertising is a means of promoting products and services using the Internet. The revenue (in billions of dollars) for online advertising for the years 2003 through a projected 2010 is given by \( y = 3.3x - 3.1 \), where \( x \) is the number of years after 2000.

a. Use this equation to complete the ordered pair \((9, \_\_)\).

b. Write a sentence explaining the meaning of the answer to part (a).
Recall that the form \( y = mx + b \) is appropriately called the slope-intercept form of a linear equation.

\[
\begin{align*}
\text{slope-intercept form} & \quad y = mx + b \\
\text{y-intercept} & \quad (0, b)
\end{align*}
\]

**Slope-Intercept Form**

When a linear equation in two variables is written in slope-intercept form,

\[
y = mx + b
\]

then \( m \) is the slope of the line and \((0, b)\) is the y-intercept of the line.

**OBJECTIVE 1** Using the slope-intercept form to write an equation. As we know from the previous section, writing an equation in slope-intercept form is a way to find the slope and y-intercept of its graph. The slope-intercept form can be used to write the equation of a line when we know its slope and y-intercept.

**EXAMPLE 1** Find an equation of the line with y-intercept \((0, -3)\) and slope of \(\frac{1}{4}\).

**Solution** We are given the slope and the y-intercept. We let \( m = \frac{1}{4} \) and \( b = -3 \) and write the equation in slope-intercept form, \( y = mx + b \).

\[
y = \frac{1}{4}x - 3
\]

**PRACTICE** Find an equation of the line with y-intercept \((0, 7)\) and slope of \(\frac{1}{2}\).

**OBJECTIVE 2** Using the slope-intercept form to graph an equation. We also can use the slope-intercept form of the equation of a line to graph a linear equation.

**EXAMPLE 2** Use the slope-intercept form to graph the equation

\[
y = \frac{3}{5}x - 2
\]

**Solution** Since the equation \( y = \frac{3}{5}x - 2 \) is written in slope-intercept form \( y = mx + b \), the slope of its graph is \( \frac{3}{5} \) and the y-intercept is \((0, -2)\). To graph this equation, we begin by plotting the point \((0, -2)\).

From this point, we can find another point of the graph by using the slope \(\frac{3}{5}\) and recalling that slope is rise over run. We start at the y-intercept and move 3 units up since the numerator of the slope is 3; then we move 5 units to the right since the denominator of the slope is 5. We stop at the point \((5, 1)\). The line through \((0, -2)\) and \((5, 1)\) is the graph of \( y = \frac{3}{5}x - 2 \).

**PRACTICE** Graph \( y = \frac{2}{3}x - 5 \).
Example 3

Use the slope-intercept form to graph the equation \(4x + y = 1\).

Solution

First we write the given equation in slope-intercept form.

\[
\begin{align*}
4x + y &= 1 \\
y &= -4x + 1
\end{align*}
\]

The graph of this equation will have slope \(-4\) and \(y\)-intercept \((0, 1)\). To graph this line, we first plot the point \((0, 1)\). To find another point of the graph, we use the slope which can be written as \(-4\left(\frac{4}{1}\right)\). We start at the point \((0, 1)\) and move 4 units down (since the numerator of the slope is \(-4\)), and then 1 unit to the right (since the denominator of the slope is 1).

We arrive at the point \((1, -3)\). The line through \((0, 1)\) and \((1, -3)\) is the graph of \(4x + y = 1\).

Practice 3

Use the slope-intercept form to graph the equation \(3x - y = 2\).

Objective 3

Writing an equation given slope and a point. Thus far, we have seen that we can write an equation of a line if we know its slope and \(y\)-intercept. We can also write an equation of a line if we know its slope and any point on the line. To see how we do this, let \(m\) represent slope and \((x_1, y_1)\) represent the point on the line. Then if \((x, y)\) is any other point of the line, we have that

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - y_1 &= m(x - x_1) \\
&= \text{slope}
\end{align*}
\]

This is the point-slope form of the equation of a line.

Point-Slope Form of the Equation of a Line

The point-slope form of the equation of a line is

\[
y - y_1 = m(x - x_1)
\]

where \(m\) is the slope of the line and \((x_1, y_1)\) is a point on the line.

Example 4

Find an equation of the line with slope \(-2\) that passes through \((-1, 5)\). Write the equation in slope-intercept form, \(y = mx + b\), and in standard form, \(Ax + By = C\).

Solution

Since the slope and a point on the line are given, we use point-slope form \(y - y_1 = m(x - x_1)\) to write the equation. Let \(m = -2\) and \((-1, 5) = (x_1, y_1)\).

\[
\begin{align*}
y - 5 &= -2(x - (-1)) \\
y - 5 &= -2(x + 1) \\
y - 5 &= -2x - 2
\end{align*}
\]

Helpful Hint

In Example 3, if we interpret the slope of \(-4\) as \(-\frac{4}{1}\), we arrive at \((-1, 5)\) for a second point. Notice that this point is also on the line.
To write the equation in slope-intercept form, \( y = mx + b \), we simply solve the equation for \( y \). To do this, we add 5 to both sides.

\[
y - 5 = -2x - 2
\]

Slope-intercept form.

\[
y = -2x + 3
\]

Add 2x to both sides and we have standard form.

**OBJECTIVE 4** Writing an equation given two points. We can also find the equation of a line when we are given any two points of the line.

**EXAMPLE 5** Find an equation of the line through \((2, 5)\) and \((-3, 4)\). Write the equation in standard form.

**Solution** First, use the two given points to find the slope of the line.

\[
m = \frac{4 - 5}{-3 - 2} = \frac{-1}{-5} = \frac{1}{5}
\]

Next we use the slope \(\frac{1}{5}\) and either one of the given points to write the equation in point-slope form. We use \((2, 5)\). Let \(x_1 = 2\), \(y_1 = 5\), and \(m = \frac{1}{5}\).

\[
y - y_1 = m(x - x_1) \quad \text{Use point-slope form.}
\]

\[
y - 5 = \frac{1}{5}(x - 2)
\]

Let \(x_1 = 2\), \(y_1 = 5\), and \(m = \frac{1}{5}\).

\[
5(y - 5) = 5 \cdot \frac{1}{5}(x - 2) \quad \text{Multiply both sides by 5 to clear fractions.}
\]

\[
5y - 25 = x - 2
\]

Subtract 2 from both sides.

\[
x - 5y - 25 = x - 2
\]

Use the distributive property and simplify.

\[
x - 5y = 23
\]

Add 25 to both sides.

**OBJECTIVE 5** Finding equations of vertical and horizontal lines. Recall from Section 3.3 that:

- **Vertical Line**: \( x = c \) with \((c, 0)\)
- **Horizontal Line**: \( y = c \) with \((0, c)\)
EXAMPLE 6  Find an equation of the vertical line through \((-1, 5)\).

Solution  The equation of a vertical line can be written in the form \(x = c\), so an equation for a vertical line passing through \((-1, 5)\) is \(x = -1\).

PRACTICE  Find an equation of the vertical line through \((3, -2)\).

EXAMPLE 7  Find an equation of the line parallel to the line \(y = 5\) and passing through \((-2, -3)\).

Solution  Since the graph of \(y = 5\) is a horizontal line, any line parallel to it is also horizontal. The equation of a horizontal line can be written in the form \(y = c\). An equation for the horizontal line passing through \((-2, -3)\) is \(y = -3\).

PRACTICE  Find an equation of the line parallel to the line \(y = -2\) and passing through \((4, 3)\).

Note: Further discussion of parallel and perpendicular lines is in Section 8.1.

OBJECTIVE 6  Using the point-slope form to solve problems. Problems occurring in many fields can be modeled by linear equations in two variables. The next example is from the field of marketing and shows how consumer demand of a product depends on the price of the product.

EXAMPLE 8  Predicting the Sales of T-Shirts

A web-based T-shirt company has learned that by pricing a clearance-sale T-shirt at $6, sales will reach 2000 T-shirts per day. Raising the price to $8 will cause the sales to fall to 1500 T-shirts per day.

a. Assume that the relationship between sales price and number of T-shirts sold is linear and write an equation describing this relationship. Write the equation in slope-intercept form.

b. Predict the daily sales of T-shirts if the price is $7.50.

Solution

a. First, use the given information and write two ordered pairs. Ordered pairs will be in the form \((\text{sales price}, \text{number sold})\) so that our ordered pairs are \((6, 2000)\) and \((8, 1500)\).
(8, 1500). Use the point-slope form to write an equation. To do so, we find the slope of the line that contains these points.

\[ m = \frac{2000 - 1500}{6 - 8} = \frac{500}{-2} = -250 \]

Next, use the slope and either one of the points to write the equation in point-slope form. We use (6, 2000).

\[
\begin{align*}
y - y_1 &= m(x - x_1) & \text{Use point-slope form.} \\
y - 2000 &= -250(x - 6) & \text{Let } x_1 = 6, y_1 = 2000, \text{ and } m = -250. \\
y - 2000 &= -250x + 1500 & \text{Use the distributive property.} \\
y &= -250x + 3500 & \text{Write in slope-intercept form.}
\end{align*}
\]

b. To predict the sales if the price is $7.50, we find \( y \) when \( x = 7.50 \).

\[
\begin{align*}
y &= -250x + 3500 \\
y &= -250(7.50) + 3500 & \text{Let } x = 7.50. \\
y &= -1875 + 3500 \\
y &= 1625
\end{align*}
\]

If the price is $7.50, sales will reach 1625 T-shirts per day.

**PRACTICE**

The new Camelot condos were selling at a rate of 30 per month when they were priced at $150,000 each. Lowering the price to $120,000 caused the sales to rise to 50 condos per month.

a. Assume that the relationship between number of condos sold and price is linear, and write an equation describing this relationship. Write the equation in slope-intercept form.

b. What should the condos be priced at if the developer wishes to sell 60 condos per month?

The preceding example may also be solved by using ordered pairs of the form (number sold, sales price).

**Forms of Linear Equations**

- **Standard form** of a linear equation.
  - \( Ax + By = C \)
  - \( A \) and \( B \) are not both 0.

- **Slope-intercept form** of a linear equation.
  - The slope is \( m \) and the \( y \)-intercept is \((0, b)\).
  - \( y = mx + b \)

- **Point-slope form** of a linear equation.
  - The slope is \( m \) and \((x_1, y_1)\) is a point on the line.
  - \( y - y_1 = m(x - x_1) \)

- **Horizontal line**
  - The slope is 0 and the \( y \)-intercept is \((0, c)\).
  - \( y = c \)

- **Vertical line**
  - The slope is undefined and the \( x \)-intercept is \((c, 0)\).
  - \( x = c \)

**Parallel and Perpendicular Lines**

Nonvertical parallel lines have the same slope.

The product of the slopes of two nonvertical perpendicular lines is \(-1\).
Graphing Calculator Explorations

A grapher is a very useful tool for discovering patterns. To discover the change in the graph of a linear equation caused by a change in slope, try the following. Use a standard window and graph a linear equation in the form \( y = mx + b \). Recall that the graph of such an equation will have slope \( m \) and \( y \)-intercept \( b \).

First graph \( y = x + 3 \). To do so, press the key and enter \( Y_1 = x + 3 \).

Notice that this graph has slope 1 and that the \( y \)-intercept is 3. Next, on the same set of axes, graph \( y = 2x + 3 \) and \( y = 3x + 3 \) by pressing and entering

\[
Y_2 = 2x + 3 \\
Y_3 = 3x + 3
\]

Notice the difference in the graph of each equation as the slope changes from 1 to 2 to 3. How would the graph of \( y = 5x + 3 \) appear? To see the change in the graph caused by a change in negative slope, try graphing \( y = -x + 3 \), \( y = -2x + 3 \), and \( y = -3x + 3 \) on the same set of axes.

Use a grapher to graph the following equations. For each exercise, graph the first equation and use its graph to predict the appearance of the other equations. Then graph the other equations on the same set of axes and check your prediction.

1. \( y = x; y = 6x, y = -6x \)
2. \( y = -x; y = -5x, y = -10x \)
3. \( y = \frac{1}{2}x + 2; y = \frac{3}{4}x + 2, y = x + 2 \)
4. \( y = x + 1; y = \frac{5}{4}x + 1, y = \frac{5}{2}x + 1 \)
5. \( y = -7x + 5; y = 7x + 5 \)
6. \( y = 3x - 1; y = -3x - 1 \)

VOCABULARY & READINESS CHECK

Use the choices below to fill in each blank. Some choices may be used more than once and some not at all.

- \( b \) (\( y_1, x_1 \)) point-slope
- \( m \) (\( x_1, y_1 \)) slope-intercept
- vertical
- standard

1. The form \( y = mx + b \) is called \underline{standard} form. When a linear equation in two variables is written in this form, \( \underline{m} \) is the slope of its graph and \( (0, \underline{b}) \) is its \( y \)-intercept.

2. The form \( y - y_1 = m(x - x_1) \) is called \underline{point-slope} form. When a linear equation in two variables is written in this form, \( \underline{m} \) is the slope of its graph and \( (x_1, y_1) \) is a point on the graph.

For Exercises 3, 4, and 7, identify the form in which the linear equation in two variables is written. For Exercises 5 and 6, identify the appearance of the graph of the equation.

3. \( y - 7 = 4(x + 3); \underline{point-slope} \) form
4. \( 5x - 9y = 11; \underline{standard} \) form
5. \( y = \frac{1}{2}; \underline{horizontal} \) line
6. \( x = -17; \underline{vertical} \) line
7. \( y = \frac{2}{3}x - \frac{1}{3}; \underline{point-slope} \) form
3.5 | EXERCISE SET

Write an equation of the line with each given slope, \( m \), and \( y \)-intercept, \( (0, b) \). See Example 1.

1. \( m = 5, b = 3 \)
2. \( m = -3, b = -3 \)
3. \( m = -4, b = \frac{1}{6} \)
4. \( m = 2, b = \frac{3}{4} \)
5. \( m = \frac{2}{3}, b = 0 \)
6. \( m = -\frac{4}{5}, b = 0 \)
7. \( m = 0, b = -8 \)
8. \( m = 0, b = -2 \)
9. \( m = -\frac{1}{5}, b = \frac{1}{9} \)
10. \( m = \frac{1}{2}, b = -\frac{1}{3} \)

Use the slope-intercept form to graph each equation. See Examples 2 and 3.

11. \( y = 2x + 1 \)
12. \( y = -4x - 1 \)
13. \( y = \frac{2}{3}x + 5 \)
14. \( y = \frac{1}{4}x - 3 \)
15. \( y = -5x \)
16. \( y = -6x \)
17. \( 4x + y = 6 \)
18. \( -3x + y = 2 \)
19. \( 4x - 7y = -14 \)
20. \( 3x - 4y = 4 \)
21. \( x = \frac{5}{4}y \)
22. \( x = \frac{3}{2}y \)

Find an equation of each line with the given slope that passes through the given point. Write the equation in the form \( Ax + By = C \). See Example 4.

23. \( m = 6; (2, 2) \)
24. \( m = 4; (1, 3) \)
25. \( m = -8; (-1, -5) \)
26. \( m = -2; (-11, -12) \)
27. \( m = \frac{3}{2}; (5, -6) \)
28. \( m = \frac{2}{3}; (-8, 9) \)
29. \( m = \frac{1}{2}; (-3, 0) \)
30. \( m = -\frac{1}{5}; (4, 0) \)

Find an equation of the line passing through each pair of points. Write the equation in the form \( Ax + By = C \). See Example 5.

31. \((3, 2)\) and \((5, 6)\)
32. \((6, 2)\) and \((8, 8)\)
33. \((-1, 3)\) and \((-2, -5)\)
34. \((-4, 0)\) and \((6, -1)\)
35. \((2, 3)\) and \((-1, -1)\)
36. \((7, 10)\) and \((-1, -1)\)
37. \((0, 0)\) and \(\left(\frac{1}{8}, \frac{1}{15}\right)\)
38. \((0, 0)\) and \(\left(-\frac{1}{2}, \frac{1}{3}\right)\)

Find an equation of each line. See Example 6.

39. Vertical line through \((0, 2)\)
40. Horizontal line through \((1, 4)\)
41. Horizontal line through \((-1, 3)\)
42. Vertical line through \((-1, 3)\)
43. Vertical line through \(\left(-\frac{7}{3}, \frac{2}{3}\right)\)
44. Horizontal line through \(\left(\frac{2}{3}, 0\right)\)

Find an equation of each line. See Example 7.

45. Parallel to \( y = 5 \), through \((1, 2)\)
46. Perpendicular to \( y = 5 \), through \((1, 2)\)
47. Perpendicular to \( x = -3 \), through \((-2, 5)\)
48. Parallel to \( y = -4 \), through \((0, -3)\)
49. Parallel to \( x = 0 \), through \((6, -8)\)
50. Perpendicular to \( x = 7 \), through \((-5, 0)\)

MIXED PRACTICE

See Examples 1 through 7. Find an equation of each line described. Write each equation in slope-intercept form (solved for \( y \)) when possible.

51. With slope \( \frac{1}{2} \), through \((0, \frac{5}{3})\)
52. With slope \( \frac{5}{7} \), through \((0, -3)\)
53. Through \((10, 7)\) and \((7, 10)\)
54. Through \((5, -6)\) and \((-6, 5)\)
55. With undefined slope, through \(\left(-\frac{3}{4}, 1\right)\)
56. With slope 0, through \((6.7, 12.1)\)
57. Slope 1, through \((-7, 9)\)
58. Slope 5, through \((6, -8)\)
59. Slope \(-5\), \(y\)-intercept \((0, 7)\)
60. Slope \(-2\), \(y\)-intercept \((0, -4)\)
61. Through \((6, 7)\), parallel to the \(x\)-axis
62. Through \((1, -5)\), parallel to the \(y\)-axis
63. Through \((2, 3)\) and \((0, 0)\)
64. Through \((4, 7)\) and \((0, 0)\)
65. Through \((-2, -3)\), perpendicular to the y-axis
66. Through \((0, 12)\), perpendicular to the x-axis
67. Slope \(-\frac{4}{7}\), through \((-1, -2)\)
68. Slope \(-\frac{3}{5}\), through \((4, 4)\)

Solve. Assume each exercise describes a linear relationship. Write the equations in slope-intercept form. See Example 8.

69. A rock is dropped from the top of a 400-foot cliff. After 1 second, the rock is traveling 32 feet per second. After 3 seconds, the rock is traveling 96 feet per second.

\[ \text{Distance} = \text{Speed} \times \text{Time} \]

\[ \text{Speed} = \frac{\text{Distance}}{\text{Time}} \]

\[ \text{Speed} = \frac{32}{1} = 32 \text{ ft/sec} \]

\[ \text{Speed} = \frac{96}{3} = 32 \text{ ft/sec} \]

70. A Hawaiian fruit company is studying the sales of a pineapple sauce to see if this product is to be continued. At the end of its first year, profits on this product amounted to $30,000. At the end of the fourth year, profits were $66,000.

a. Assume that the relationship between years on the market and profit is linear and write an equation describing this relationship. Use ordered pairs of the form \((\text{years}, \text{profit})\).

b. Use this equation to predict the profit at the end of 7 years.

71. In January 2007, there were 71,000 registered gasoline-electric hybrid cars in the United States. In 2004, there were only 29,000 registered gasoline-electric hybrids. (Source: U.S. Energy Information Administration)

a. Write an equation describing the relationship between time and number of registered gasoline-hybrid cars. Use ordered pairs of the form \((\text{years}, \text{number of cars})\).

b. Use this equation to predict the number of gasoline-electric hybrids in the year 2010.

72. In 2006, there were 935 thousand eating establishments in the United States. In 1996, there were 457 thousand eating establishments. (Source: National Restaurant Association)

a. Write an equation describing the relationship between time and number of eating establishments. Use ordered pairs of the form \((\text{years past 1996}, \text{number of eating establishments in thousands})\).

b. Use this equation to predict the number of eating establishments in 2010.

73. In 2006, the U.S. population per square mile of land area was 85. In 2000, the person per square mile population was 79.6.

a. Write an equation describing the relationship between year and persons per square mile. Use ordered pairs of the form \((\text{years past 2000}, \text{persons per square mile})\).

b. Use this equation to predict the person per square mile population in 2010.

74. In 2001, there were a total of 152 thousand apparel and accessory stores. In 2005, there were a total of 150 thousand apparel and accessory stores. (Source: U.S. Bureau of the Census. County Business Patterns, annual)

a. Write an equation describing this relationship. Use ordered pairs of the form \((\text{years past 2001}, \text{numbers of stores in thousand})\).

b. Use this equation to predict the number of apparel and accessory stores in 2011.

75. The birth rate in the United States in 1996 was 14.7 births per thousand population. In 2006, the birth rate was 14.14 births per thousand. (Source: Department of Health and Human Services, National Center for Health Statistics)

a. Write two ordered pairs of the form \((\text{years after 1996}, \text{birth rate per thousand population})\).

b. Assume that the relationship between years after 1996 and birth rate per thousand is linear over this period. Use the ordered pairs from part (a) to write an equation of the line relating years to birth rate.

c. Use the linear equation from part (b) to estimate the birth rate in the United States in the year 2016.
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76. In 2002, crude oil production by OPEC countries was about 28.7 million barrels per day. In 2007, crude oil production had risen to about 34.5 million barrels per day. (Source: OPEC)
   a. Write two ordered pairs of the form (years after 2002, crude oil production) for this situation.
   b. Assume that crude oil production is linear between the years 2002 and 2007. Use the ordered pairs from part (a) to write an equation of the line relating year and crude oil production.
   c. Use the linear equation from part (b) to estimate the crude oil production by OPEC countries in 2004.

77. Better World Club is a relatively new automobile association which prides itself on its “green” philosophy. In 2003, the membership totaled 5 thousand. By 2006, there were 20 thousand members of this ecologically minded club. (Source: Better World Club)
   a. Write two ordered pairs of the form (years after 2003, membership in thousands)
   b. Assume that the membership is linear between the years 2003 and 2006. Use the ordered pairs from part (a) to write an equation of the line relating year and Better World membership.
   c. Use the linear equation from part (b) to predict the Better World Club membership in 2012.

78. In 2002, 9.9 million electronic bill statements were delivered and payment occurred. In 2005, that number rose to 26.9 million. (Source: Forrester Research)
   a. Write two ordered pairs of the form (years after 2002, millions of electronic bills).
   b. Assume that this method of delivery and payment between the years 2002 and 2005 is linear. Use the ordered pairs from part (a) to write an equation of the line relating year and number of electronic bills.
   c. Use the linear equation from part (b) to predict the number of electronic bills to be delivered and paid in 2011.

CONCEPT EXTENSIONS

83. Given the equation of a nonvertical line, explain how to find the slope without finding two points on the line.
84. Given two points on a nonvertical line, explain how to use the point-slope form to find the equation of the line.
85. Write an equation in standard form of the line that contains the point (−1, 2) and is parallel to the line $y = 3x - 1$.
86. Write an equation in standard form of the line that contains the point (4, 0) and is perpendicular to the line $y = -2x + 3$.
87. Write an equation in standard form of the line that contains the point (3, −5) and is parallel to the line $3x + 2y = 7$.
88. Write an equation in standard form of the line that contains the point (−2, 4) and is perpendicular to the line $x + 3y = 6$.

REVIEW AND PREVIEW

Find the value of $x^2 - 3x + 1$ for each given value of $x$. See Section 1.7.

79. 2  80. 5  81. −1  82. −3
OBJECTIVE 1 Identifying relations, domains, and ranges. In previous sections, we have discussed the relationships between two quantities. For example, the relationship between the length of the side of a square \(x\) and its area \(y\) is described by the equation \(y = x^2\). Ordered pairs can be used to write down solutions of this equation. For example, \((2, 4)\) is a solution of \(y = x^2\), and this notation tells us that the \(x\)-value 2 is related to the \(y\)-value 4 for this equation. In other words, when the length of the side of a square is 2 units, its area is 4 square units.

Examples of Relationships Between Two Quantities

<table>
<thead>
<tr>
<th>Area of Square: (y = x^2)</th>
<th>Equation of Line: (y = x + 2)</th>
<th>Online Advertising Revenue</th>
</tr>
</thead>
</table>

Some Ordered Pairs

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
<th>Year</th>
<th>Billions of Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>-3</td>
<td>-1</td>
<td>2006</td>
<td>16.7</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>0</td>
<td>2</td>
<td>2007</td>
<td>20.3</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>2</td>
<td>4</td>
<td>2008</td>
<td>23.5</td>
</tr>
<tr>
<td>12</td>
<td>144</td>
<td>9</td>
<td>11</td>
<td>2009</td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2010</td>
<td>29.4</td>
</tr>
</tbody>
</table>

A set of ordered pairs is called a relation. The set of all \(x\)-coordinates is called the domain of a relation, and the set of all \(y\)-coordinates is called the range of a relation. Equations such as \(y = x^2\) are also called relations since equations in two variables define a set of ordered pair solutions.

**EXAMPLE 1** Find the domain and the range of the relation \{\((0, 2), (3, 3), (-1, 0), (5, -2)\)\}.

**Solution** The domain is the set of all \(x\)-values or \{-1, 0, 3\}, and the range is the set of all \(y\)-values, or \{-2, 0, 2, 3\}.

**PRACTICE 1** Find the domain and the range of the relation \{\((1, 3), (5, 0), (0, -2), (5, 4)\)\}.

OBJECTIVE 2 Identifying functions. Some relations are also functions.

**Function**
A function is a set of ordered pairs that assigns to each \(x\)-value exactly one \(y\)-value.
EXAMPLE 2 Which of the following relations are also functions?

a. \{(-1, 1), (2, 3), (7, 3), (8, 6)\}  
b. \{(0, -2), (1, 5), (0, 3), (7, 7)\}

Solution

a. Although the ordered pairs (2, 3) and (7, 3) have the same y-value, each x-value is assigned to only one y-value so this set of ordered pairs is a function.

b. The x-value 0 is assigned to two y-values, -2 and 3, so this set of ordered pairs is not a function.

PRACTICE 2 Which of the following relations are also functions?

a. \{(4, 1)(3, -2)(8, 5)(-5, 3)\}  
b. \{(1, 2)(-4, 3)(0, 8)(1, 4)\}

Relations and functions can be described by a graph of their ordered pairs.

EXAMPLE 3 Which graph is the graph of a function?

a.  
b.

Solution

a. This is the graph of the relation \{(-4, -2), (-2, -1)(-1, -1), (1, 2)\}. Each x-coordinate has exactly one y-coordinate, so this is the graph of a function.

b. This is the graph of the relation \{(-2, -3), (1, 2), (1, 3), (2, -1)\}. The x-coordinate 1 is paired with two y-coordinates, 2 and 3, so this is not the graph of a function.

PRACTICE 3 Which graph is the graph of a function?

a.  
b.

OBJECTIVE 3 Using the vertical line test. The graph in Example 3(b) was not the graph of a function because the x-coordinate 1 was paired with two y-coordinates, 2 and 3. Notice that when an x-coordinate is paired with more than one y-coordinate, a vertical line can be drawn that will intersect the graph at more than one point. We can use this fact to determine whether a relation is also a function. We call this the vertical line test.
Section 3.6 Functions

Vertical Line Test

If a vertical line can be drawn so that it intersects a graph more than once, the graph is not the graph of a function.

This vertical line test works for all types of graphs on the rectangular coordinate system.

**EXAMPLE 4** Use the vertical line test to determine whether each graph is the graph of a function.

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
<th>c.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Graph A]</td>
<td>![Graph B]</td>
<td>![Graph C]</td>
<td>![Graph D]</td>
</tr>
</tbody>
</table>

**Solution**

a. This graph is the graph of a function since no vertical line will intersect this graph more than once.

b. This graph is also the graph of a function; no vertical line will intersect it more than once.

c. This graph is not the graph of a function. Vertical lines can be drawn that intersect the graph in two points. An example of one is shown.

d. This graph is not the graph of a function. A vertical line can be drawn that intersects this line at every point.

**Practice**

4 Use the vertical line test to determine whether each graph is the graph of a function.

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
<th>c.</th>
<th>d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Graph A]</td>
<td>![Graph B]</td>
<td>![Graph C]</td>
<td>![Graph D]</td>
</tr>
</tbody>
</table>

Recall that the graph of a linear equation is a line, and a line that is not vertical will pass the vertical line test. Thus, all linear equations are functions except those of the form \( x = c \), which are vertical lines.
**Example 5** Which of the following linear equations are functions?

a. \( y = x \)  
   b. \( y = 2x + 1 \)  
   c. 5  
   d. \( x = -1 \)

**Solution**  

a, b, and c are functions because their graphs are nonvertical lines. d is not a function because its graph is a vertical line.

**Practice 5** Which of the following linear equations are functions?

a. \( y = 2x \)  
   b. \( y = -3x - 1 \)  
   c. 8  
   d. \( x = 2 \)

Examples of functions can often be found in magazines, newspapers, books, and other printed material in the form of tables or graphs such as that in Example 6.

**Example 6** The graph shows the sunrise time for Indianapolis, Indiana, for the year. Use this graph to answer the questions.

![Graph of sunrise times](image)

a. Approximate the time of sunrise on February 1.
   b. Approximately when does the sun rise at 5 a.m.?
   c. Is this the graph of a function?

**Solution**

a. To approximate the time of sunrise on February 1, we find the mark on the horizontal axis that corresponds to February 1. From this mark, we move vertically upward until the graph is reached. From that point on the graph, we move horizontally to the left until the vertical axis is reached. The vertical axis there reads 7 a.m.
Section 3.6 Functions

b. To approximate when the sun rises at 5 a.m., we find 5 a.m. on the time axis and move horizontally to the right. Notice that we will reach the graph twice, corresponding to two dates for which the sun rises at 5 a.m. We follow both points on the graph vertically downward until the horizontal axis is reached. The sun rises at 5 a.m. at approximately the end of the month of April and the middle of the month of August.

c. The graph is the graph of a function since it passes the vertical line test. In other words, for every day of the year in Indianapolis, there is exactly one sunrise time.

PRACTICE 6 The graph shows the average monthly temperature for Chicago, Illinois, for the year. Use this graph to answer the questions.

![Average Monthly Temperature Graph](image)

*a*(1 is Jan., 12 is Dec.)

a. Approximate the average monthly temperature for June.
b. Approximately when is the average monthly temperature 40°?
c. Is this the graph of a function?

OBJECTIVE 4 Using function notation. The graph of the linear equation \( y = 2x + 1 \) passes the vertical line test, so we say that \( y = 2x + 1 \) is a function. In other words, \( y = 2x + 1 \) gives us a rule for writing ordered pairs where every \( x \)-coordinate is paired with one \( y \)-coordinate.
We often use letters such as $f$, $g$, and $h$ to name functions. For example, the symbol $f(x)$ means function of $x$ and is read “$f$ of $x$.” This notation is called function notation. The equation $y = 2x + 1$ can be written as $f(x) = 2x + 1$ using function notation, and these equations mean the same thing. In other words, $y = f(x)$.

The notation $f(1)$ means to replace $x$ with 1 and find the resulting $y$ or function value. Since

$$f(x) = 2x + 1$$

then

$$f(1) = 2(1) + 1 = 3$$

This means that, when $x = 1$, $y$ or $f(x) = 3$, and we have the ordered pair $(1, 3)$. Now let’s find $f(2), f(0)$, and $f(-1)$.

$$f(x) = 2x + 1$$

Ordered Pair: $(2, 5)$

$$f(2) = 2(2) + 1 = 5$$

Ordered Pair: $(0, 1)$

$$f(0) = 2(0) + 1 = 1$$

Ordered Pair: $(-1, -1)$

$$f(-1) = 2(-1) + 1 = -1$$

**Example 7** Given $g(x) = x^2 - 3$, find the following. Then write down the corresponding ordered pairs generated.

a. $g(2)$  

b. $g(-2)$  

c. $g(0)$  

**Solution**

$$g(x) = x^2 - 3$$

$g(2) = 2^2 - 3 = 4 - 3 = 1$  

$g(-2) = (-2)^2 - 3 = 4 - 3 = 1$  

$g(0) = 0^2 - 3 = 0 - 3 = -3$

Ordered Pairs: $g(2) = 1$ gives $(2, 1)$  

$g(-2) = 1$ gives $(-2, 1)$  

$g(0) = -3$ gives $(0, -3)$

**Practice** 7 Given $h(x) = x^2 + 5$, find the following. Then write the corresponding ordered pairs generated.

a. $h(2)$  

b. $h(-5)$  

c. $h(0)$  

We now practice finding the domain and the range of a function. The domain of our functions will be the set of all possible real numbers that $x$ can be replaced by. The range is the set of corresponding $y$-values.
EXAMPLE 8  Find the domain of each function.

a. \( g(x) = \frac{1}{x} \)  
   \[ \text{Domain: All real numbers except 0} \]

b. \( f(x) = 2x + 1 \)  
   \[ \text{Domain: All real numbers} \]

Solution

a. Recall that we cannot divide by 0 so that the domain of \( g(x) \) is the set of all real numbers except 0. In interval notation, we can write \((-\infty, 0) \cup (0, \infty)\).

b. In this function, \( x \) can be any real number. The domain of \( f(x) \) is the set of all real numbers, or \((-\infty, \infty)\) in interval notation.

PRACTICE 8  Find the domain of each function.

a. \( h(x) = 6x + 3 \)  
   \[ \text{Domain: All real numbers} \]

b. \( f(x) = \frac{1}{x^2} \)  
   \[ \text{Domain: All real numbers except 0} \]

Concept Check

Suppose that the value of \( f \) is \(-7\) when the function is evaluated at 2. Write this situation in function notation.

EXAMPLE 9  Find the domain and the range of each function graphed. Use interval notation.

a. \[
\begin{align*}
&x \quad y \\
&-4 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \\
&-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \quad 4 \\
&(-4, -1) \quad (1, 3) \quad (4, 5)
\end{align*}
\]
   \[ \text{Range: } [-1, 5] \]

b. \[
\begin{align*}
&x \quad y \\
&-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
&-4 \quad -3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \quad 4 \\
&(3, -2)
\end{align*}
\]
   \[ \text{Range: } [-2, 5] \]

Solution

a. \[
\begin{align*}
&x \quad y \\
&-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \\
&-4 \quad -3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \\
&(3, -2) \quad (1, 3) \quad (4, 5)
\end{align*}
\]
   \[ \text{Domain: } [-3, 3] \]

b. \[
\begin{align*}
&x \quad y \\
&-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
&-4 \quad -3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3 \quad 4 \\
&(3, -2)
\end{align*}
\]
   \[ \text{Domain: } (-\infty, \infty) \]

Answer to Concept Check:

\( f(2) = -7 \)
CHAPTER 3  Graphs and Introduction to Functions

Find the domain and the range of each function graphed. Use interval notation.

a.  

\[(4, 3)\]  

b.  

\[(-4, 3)\]  

\[(-6, -2)\]

### VOCABULARY & READINESS CHECK

Use the choices below to fill in each blank. Some choices may not be used.

- \[x = c\]  
- horizontal  
- domain  
- relation  
- \[(7, 3)\]  
- \[(-\infty, 5]\]  
- \[y = c\]  
- vertical  
- range  
- function  
- \[(3, 7)\]  
- \[(-\infty, \infty)\]

1. A set of ordered pairs is called a ________.
2. A set of ordered pairs that assigns to each \(x\)-value exactly one \(y\)-value is called a ________.
3. The set of all \(y\)-coordinates of a relation is called the ________.
4. The set of all \(x\)-coordinates of a relation is called the ________.
5. All linear equations are functions except those whose graphs are ________ lines.
6. All linear equations are functions except those whose equations are of the form ________.
7. If \(f(3) = 7\), the corresponding ordered pair is ________.
8. The domain of \(f(x) = x + 5\) is ________.

### 3.6 EXERCISE SET

Find the domain and the range of each relation. See Example 1.

1. \{(2, 4), (0, 0), (−7, 10), (10, −7)\}
2. \{(3, −6), (1, 4), (−2, −2)\}
3. \{(0, −2), (1, −2), (5, −2)\}
4. \{(5, 0), (5, −3), (5, 4), (5, 3)\}

Determine whether each relation is also a function. See Example 2.

5. \{(1, 1), (2, 2), (−3, −3), (0, 0)\}
6. \{(11, 6), (−1, −2), (0, 0), (3, −2)\}
7. \{−1, 0\}, \{−1, 6\}, \{−1, 8\}
8. \{(1, 2), (3, 2), (1, 4)\}

### MIXED PRACTICE

Determine whether each graph is the graph of a function. See Examples 3 and 4.

9.  

\[(-5, 3)\]  

\[(-4, 2)\]  

10.  

\[(-5, 3)\]  

\[(-4, 2)\]
Section 3.6 Functions

Decide whether the equation describes a function. See Example 5.

17. \( y = x + 1 \) 
18. \( y = x - 1 \) 
19. \( y - x = 7 \) 
20. \( 2x - 3y = 9 \) 
21. \( y = 6 \) 
22. \( x = 3 \) 
23. \( x = -2 \) 
24. \( y = -9 \) 
25. \( x = y^2 \) 
26. \( y = x^2 - 3 \)

The graph shows the sunset times for Seward, Alaska. Use this graph to answer Exercises 27 through 32. See Example 6.

27. Approximate the time of sunset on June 1.
28. Approximate the time of sunset on November 1.
29. Approximate the date(s) when the sunset is at 3 p.m.
30. Approximate the date(s) when the sunset is at 9 p.m.
31. Is this graph the graph of a function? Why or why not?
32. Do you think a graph of sunset times for any location will always be a function? Why or why not?

This graph shows the U.S. hourly minimum wage for each year shown. Use this graph to answer Exercises 33 through 38. See Example 6.

33. Approximate the minimum wage before October, 1996.
34. Approximate the minimum wage in 2006.
35. Approximate the year when the minimum wage will increase to over $7.00 per hour.
36. According to the graph, what hourly wage was in effect for the greatest number of years?

37. Is this graph the graph of a function? Why or why not?

38. Do you think that a similar graph of your hourly wage on January 1 of every year (whether you are working or not) will be the graph of a function? Why or why not?

Find \( f(-2), f(0), \) and \( f(3) \) for each function. See Example 7.

39. \( f(x) = 2x - 5 \)
40. \( f(x) = 3 - 7x \)
41. \( f(x) = x^2 + 2 \)
42. \( f(x) = x^2 - 4 \)
43. \( f(x) = 3x \)
44. \( f(x) = -3x \)
45. \( f(x) = |x| \)
46. \( f(x) = |2 - x| \)

Find \( h(-1), h(0), \) and \( h(4) \) for each function. See Example 7.

47. \( h(x) = -5x \)
48. \( h(x) = -3x \)
49. \( h(x) = 2x^2 + 3 \)
50. \( h(x) = 3x^2 \)

For each given function value, write a corresponding ordered pair.

51. \( f(3) = 6 \)
52. \( f(7) = -2 \)
53. \( g(0) = -\frac{1}{2} \)
54. \( g(0) = \frac{7}{8} \)
55. \( h(-2) = 9 \)
56. \( h(-10) = 1 \)

Find the domain of each function. See Example 8.

57. \( f(x) = 3x - 7 \)
58. \( g(x) = 5 - 2x \)
59. \( h(x) = \frac{1}{x + 5} \)
60. \( f(x) = \frac{1}{x - 6} \)
61. \( g(x) = |x + 1| \)
62. \( h(x) = |2x| \)

Find the domain and the range of each relation graphed. See Example 9.

63. 

64. 

65. 

66. 

67. 

68. 

69. 

70. 

71. 

72. 

73. If a function \( f \) is evaluated at \(-5\), the value of the function is 12. Write this situation using function notation.

74. Suppose \((9, 20)\) is an ordered-pair solution for the function \( g \). Write this situation using function notation.

The graph of the function, \( f \), is below. Use this graph to answer Exercises 75 through 78.

75. Write the coordinates of the lowest point of the graph.

76. Write the answer to Exercise 75 in function notation.
77. An x-intercept of this graph is (5, 0). Write this using function notation.
78. Write the other x-intercept of this graph (see Exercise 77) using function notation.
79. Forensic scientists use the function 
   \[ H(x) = 2.59x + 47.24 \]
   to estimate the height of a woman in centimeters given the length x of her femur bone.
   
   a. Estimate the height of a woman whose femur measures 46 centimeters.
   b. Estimate the height of a woman whose femur measures 39 centimeters.
80. The dosage in milligrams \( D \) of Ivermectin, a heartworm preventive for a dog who weighs \( x \) pounds, is given by the function
   \[ D(x) = \frac{136}{25}x^2 \]
   a. Find the proper dosage for a dog that weighs 35 pounds.
   b. Find the proper dosage for a dog that weighs 70 pounds.
81. In your own words define (a) function; (b) domain; (c) range.
82. Explain the vertical line test and how it is used.
83. Since \( y = x + 7 \) is a function, rewrite the equation using function notation.

**See the example below for Exercises 84 through 87.**

Example: If \( f(x) = x^2 + 2x + 1 \), find \( f(\pi) \).

Solution:
\[
\begin{align*}
    f(x) &= x^2 + 2x + 1 \\
    f(\pi) &= \pi^2 + 2\pi + 1
\end{align*}
\]

Given the following functions, find the indicated values.

84. \( f(x) = x^2 + 7 \)
   a. \( f(2) \) 
   b. \( f(a) \)
85. \( g(x) = -3x + 12 \)
   a. \( g(s) \) 
   b. \( g(r) \)
86. \( h(x) = x^2 + 7 \)
   a. \( h(3) \) 
   b. \( h(a) \)
87. \( f(x) = x^2 - 12 \)
   a. \( f(12) \) 
   b. \( f(a) \)

**CHAPTER 3 GROUP ACTIVITY**

**Financial Analysis**

Investment analysts investigate a company’s sales, net profit, debt, and assets to decide whether investing in it is a wise choice. One way to analyze this data is to graph it and look for trends over time. Another way is to find algebraically the rate at which the data changes over time.

The following table gives the net incomes in millions of dollars for some of the leading U.S. businesses in the pharmaceutical industry for the years 2004 and 2005. In this project, you will analyze the performances of these companies and, based on this information alone, make an investment recommendation. This project may be completed by working in groups or individually.

<table>
<thead>
<tr>
<th>Pharmaceutical Industry Net Income (In Millions of Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Company</strong></td>
</tr>
<tr>
<td>Merck</td>
</tr>
<tr>
<td>Pfizer</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
</tr>
<tr>
<td>Bristol-Myers Squibb</td>
</tr>
<tr>
<td>Abbot Laboratories</td>
</tr>
<tr>
<td>Eli Lilly</td>
</tr>
<tr>
<td>Schering-Plough</td>
</tr>
<tr>
<td>Wyeth</td>
</tr>
</tbody>
</table>

*Source: The 2006 annual report for each of the companies listed.*

1. Scan the table. Did any of the companies have a loss during the years shown? If so, which company and when? What does this mean?
2. Write the data for each company as two ordered pairs of the form (year, net income). Assuming that the trends in net income are linear, use graph paper to graph the line represented by the ordered pairs for each company. Describe the trends shown by each graph.
3. Find the slope of the line for each company.
4. Which of the lines, if any, have positive slopes? What does that mean in this context? Which of the lines have negative slopes? What does that mean in this context?
5. Of these pharmaceutical companies, which one(s) would you recommend as an investment choice? Why?
6. Do you think it is wise to make a decision after looking at only two years of net profits? What other factors do you think should be taken into consideration when making an investment choice?

(Optional) Use financial magazines, company annual reports, or online investing information to find net income information for two different years for two to four companies in the same industry. Analyze the net income and make an investment recommendation.
CHAPTER 3 VOCABULARY CHECK

Fill in each blank with one of the words listed below.

relation   function   domain  range  standard  slope-intercept
y-axis  x-axis  solution  linear  slope  point-slope
x-intercept  y-intercept  y  x

1. An ordered pair is a ________ of an equation in two variables if replacing the variables by the coordinates of the ordered pair results in a true statement.
2. The vertical number line in the rectangular coordinate system is called the ________.
3. A ________ equation can be written in the form $Ax + By = C$.
4. A(n) ________ is a point of the graph where the graph crosses the x-axis.
5. The form $Ax + By = C$ is called ________ form.
6. A(n) ________ is a point of the graph where the graph crosses the y-axis.
7. The equation $y = 7x - 5$ is written in ________ form.
8. The equation $y + 1 = 7(x - 2)$ is written in ________ form.
9. To find an x-intercept of a graph, let ________ = 0.
10. The horizontal number line in the rectangular coordinate system is called the ________.
11. To find a y-intercept of a graph, let ________ = 0.
12. The ________ of a line measures the steepness or tilt of a line.
13. A set of ordered pairs that assigns to each $x$-value exactly one $y$-value is called a ________.
14. The set of all $x$-coordinates of a relation is called the ________ of the relation.
15. The set of all $y$-coordinates of a relation is called the ________ of the relation.
16. A set of ordered pairs is called a ________.

Helpful Hint
Are you preparing for your test? Don’t forget to take the Chapter 3 Test on page 242. Then check your answers at the back of the text and use the Chapter Test Prep Video CD to see the fully worked-out solutions to any of the exercises you want to review.

CHAPTER 3 HIGHLIGHTS

DEFINITIONS AND CONCEPTS

The rectangular coordinate system consists of a plane and a vertical and a horizontal number line intersecting at their 0 coordinates. The vertical number line is called the y-axis and the horizontal number line is called the x-axis. The point of intersection of the axes is called the origin.
DEFINITIONS AND CONCEPTS

SECTION 3.1 READING GRAPHS AND THE RECTANGULAR COORDINATE SYSTEM (continued)

To plot or graph an ordered pair means to find its corresponding point on a rectangular coordinate system.

To plot or graph an ordered pair such as (3, -2), start at the origin. Move 3 units to the right and from there, 2 units down.

To plot or graph (-3, 4) start at the origin. Move 3 units to the left and from there, 4 units up.

An ordered pair is a solution of an equation in two variables if replacing the variables by the coordinates of the ordered pair results in a true statement.

If one coordinate of an ordered pair solution is known, the other value can be determined by substitution.

A linear equation in two variables is an equation that can be written in the form $Ax + By = C$ where $A$ and $B$ are not both 0. The form $Ax + By = C$ is called standard form.

To graph a linear equation in two variables, find three ordered pair solutions. Plot the solution points and draw the line connecting the points.

EXAMPLES

Determine whether (-1, 5) is a solution of $2x + 3y = 13$.

Let $x = -1, y = 5$

$2(-1) + 3(5) = 13$

$-2 + 15 = 13$

$13 = 13$

True

Complete the ordered pair solution (0, ) for the equation $x - 6y = 12$.

$x - 6y = 12$

$0 - 6y = 12$

$-6y = 12$

$y = -2$

The ordered pair solution is (0, -2).

SECTION 3.2 GRAPHING LINEAR EQUATIONS

Linear Equations

$3x + 2y = -6$

$x = -5$

$y = 3$

$y = -x + 10$

$x + y = 10$ is in standard form.

Graph $x - 2y = 5$.
An intercept of a graph is a point where the graph intersects an axis. If a graph intersects the x-axis at \( a \), then \((a, 0)\) is the x-intercept. If a graph intersects the y-axis at \( b \), then \((0, b)\) is the y-intercept.

To find the x-intercept, let \( y = 0 \) and solve for \( x \).
To find the y-intercept, let \( x = 0 \) and solve for \( y \).

The graph of \( x = c \) is a vertical line with x-intercept \((c, 0)\).
The graph of \( y = c \) is a horizontal line with y-intercept \((0, c)\).
DEFINITIONS AND CONCEPTS

SECTION 3.4 SLOPE AND RATE OF CHANGE

The slope \( m \) of the line through points \((x_1, y_1)\) and \((x_2, y_2)\) is given by
\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]
as long as \(x_2 \neq x_1\)

A horizontal line has slope 0.
The slope of a vertical line is undefined.
Nonvertical parallel lines have the same slope.
Two nonvertical lines are perpendicular if the slope of one is the negative reciprocal of the slope of the other.

SECTION 3.5 EQUATIONS OF LINES

**Slope-Intercept Form**
\[
y = mx + b
\]
\(m\) is the slope of the line.
\((0, b)\) is the \(y\)-intercept.

**Point-Slope Form**
\[
y - y_1 = m(x - x_1)
\]
\(m\) is the slope.
\((x_1, y_1)\) is a point on the line.
A set of ordered pairs is a relation. The set of all x-coordinates is called the domain of the relation and the set of all y-coordinates is called the range of the relation.

A function is a set of ordered pairs that assigns to each x-value exactly one y-value.

**Vertical Line Test**

If a vertical line can be drawn so that it intersects a graph more than once, the graph is not the graph of a function.

The symbol \( f(x) \) means function of \( x \). This notation is called function notation.

A function is a set of ordered pairs that assigns to each \( x \)-value exactly one \( y \)-value.

**CHAPTER 3 REVIEW**

**(3.1) Plot the following ordered pairs on a Cartesian coordinate system.**

1. \((-7, 0)\)
2. \((0, 4.5)\)
3. \((-2, -5)\)
4. \((1, -3)\)
5. \((0.7, 0.7)\)
6. \((-6, 4)\)

7. A local lumberyard uses quantity pricing. The table shows the price per board for different amounts of lumber purchased.

<table>
<thead>
<tr>
<th>Price per Board (in dollars)</th>
<th>Number of Boards Purchased</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00</td>
<td>1</td>
</tr>
<tr>
<td>7.50</td>
<td>10</td>
</tr>
<tr>
<td>6.50</td>
<td>25</td>
</tr>
<tr>
<td>5.00</td>
<td>50</td>
</tr>
<tr>
<td>2.00</td>
<td>100</td>
</tr>
</tbody>
</table>

**EXAMPLES**

The domain of the relation \{(0, 5), (2, 5), (4, 5), (5, -2)\} is \{0, 2, 4, 5\}. The range is \{-2, 5\}.

Which are graphs of functions?

If \( f(x) = 2x^2 + 6x - 1 \), find \( f(3) \).

\[
f(3) = 2(3)^2 + 6 \cdot 3 - 1 = 2 \cdot 9 + 18 - 1 = 18 + 18 - 1 = 35
\]

8. The table shows the annual overnight stays in national parks (Source: National Park Service)

<table>
<thead>
<tr>
<th>Year</th>
<th>Overnight Stays in National Parks (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>9.8</td>
</tr>
<tr>
<td>2002</td>
<td>15.1</td>
</tr>
<tr>
<td>2003</td>
<td>14.6</td>
</tr>
<tr>
<td>2004</td>
<td>14.0</td>
</tr>
<tr>
<td>2005</td>
<td>13.8</td>
</tr>
<tr>
<td>2006</td>
<td>13.6</td>
</tr>
</tbody>
</table>

**a.** Write each paired data as an ordered pair of the form (year, number of overnight stays).

**b.** Create a scatter diagram of the paired data. Be sure to label the axes properly.

**Determine whether each ordered pair is a solution of the given equation.**

9. \( 7x - 8y = 56; (0, 56), (8, 0) \)
10. \( -2x + 5y = 10; (-5, 0), (1, 1) \)
11. \( x = 13; (13, 5), (13, 13) \)
12. \( y = 2; (7, 2), (2, 7) \)
Complete the ordered pairs so that each is a solution of the given equation.
13. \(-2 + y = 6x; \(7, \) \) 14. \(y = 3x + 5; \(-2, -8\)\)

Complete the table of values for each given equation; then plot the ordered pairs. Use a single coordinate system for each exercise.

15. \(9 = -3x + 4y\) 16. \(y = 5\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>3</td>
<td>-7</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

17. \(x = 2y\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td></td>
<td>-5</td>
</tr>
</tbody>
</table>

18. The cost in dollars of producing \(x\) compact disk holders is given by \(y = 5x + 2000\).

a. Complete the following table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

b. Find the number of compact disk holders that can be produced for $6430.

(3.2) Graph each linear equation.

19. \(x - y = 1\) 20. \(x + y = 6\)
21. \(x - 3y = 12\) 22. \(5x - y = -8\)
23. \(x = 3y\) 24. \(y = -2x\)
25. \(2x - 3y = 6\) 26. \(4x - 3y = 12\)

27. The projected U.S. long-distance revenue (in billions of dollars) from 1999 to 2004 is given by the equation, \(y = 3x + 111\) where \(x\) is the number of years after 1999. Graph this equation and use it to estimate the amount of long-distance revenue in 2007. (Source: Giga Information Group)

(3.3) Identify the intercepts.

28. 

29. 

(3.4) Find the slope of each line.

30. 

31. 

Graph each linear equation by finding its intercepts.

32. \(x - 3y = 12\) 33. \(-4x + y = 8\)
34. \(y = -3\) 35. \(x = 5\)
36. \(y = -3x\) 37. \(x = 5y\)
38. \(x - 2 = 0\) 39. \(y + 6 = 0\)

In Exercises 42 through 45, match each line with its slope.

40. 

41. 

c. 

d. 

42. 

43. 

44. 

45. 

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e. 
\[ \begin{array}{|c|c|c|c|c|}
\hline
x & y & \hline
-1 & 2 & \hline
1 & 4 & \hline
3 & 5 & \hline
\end{array} \]

42. \( m = 0 \) 43. \( m = -1 \) 44. undefined slope 45. \( m = 3 \)

Find the slope of the line that goes through the given points.
47. (2, 5) and (6, 8) 48. (4, 7) and (1, 2) 49. (1, 3) and (−2, −9) 50. (−4, 1) and (3, −6)

Find the slope of each line.
51. \( y = 3x + 7 \) 52. \( x - 2y = 4 \) 53. \( y = -2 \) 54. \( x = 0 \)

△ Determine whether each pair of lines is parallel, perpendicular, or neither.
55. \( x - y = -6 \) \( x + y = 3 \) 56. \( 3x + y = 7 \) \( -3x - y = 10 \) 57. \( y = 4x + \frac{1}{2} \) \( 4x + 2y = 1 \) 58. \( x = 4 \) \( y = -2 \)

Find the slope of each line and write the slope as a rate of change. Don’t forget to attach the proper units.
59. The graph below shows the average monthly day care cost for a 3-year-old attending 8 hours a day, 5 days a week.

60. The graph below shows the U.S. government’s projected spending (in billions of dollars) on technology. (Some years projected.)

(3.5) Determine the slope and the \( y \)-intercept of the graph of each equation.
61. \( 3x + y = 7 \) 62. \( x - 6y = -1 \) 63. \( y = 2 \) 64. \( x = -5 \)

Write an equation of each line in slope-intercept form.
65. slope \(-5\); \( y \)-intercept \( \frac{1}{2} \) 66. slope \( \frac{2}{3} \); \( y \)-intercept 6

Use the slope-intercept form to graph each equation.
67. \( y = 3x - 1 \) 68. \( y = -3x \) 69. \( 5x - 3y = 15 \) 70. \( -x + 2y = 8 \)

Match each equation with its graph.
71. \( y = -4x \) 72. \( y = -2x + 1 \) 73. \( y = 2x - 1 \) 74. \( y = 2x \)

\[ \begin{array}{|c|c|c|c|c|}
\hline
35 & 40 & 45 & 50 & 55 \hline
\end{array} \]

\[ \begin{array}{|c|c|c|c|c|}
\hline
Year & Monthly Day Care Costs \hline
1985 & 232 \hline
1990 & \hline
1995 & \hline
2000 & \hline
2005 & \hline
2010 & \hline
\end{array} \]

Source: U.S. Senate Joint Economic Committee Fact Sheet

(2004, 46) (2009, 56.5)
Chapter 3 Review

Write an equation of each line in standard form.
75. With slope $-3$, through $(0, -5)$
76. With slope $\frac{1}{2}$, through $\left(0, -\frac{7}{2}\right)$
77. With slope $0$, through $(-2, -3)$
78. With $0$ slope, through the origin
79. With slope $=-6$, through $(2, -1)$
80. With slope $12$, through $\left(0, \frac{1}{2}\right)$
81. Through $(0, 6)$ and $(6, 0)$
82. Through $(0, -4)$ and $(-8, 0)$
83. Vertical line, through $(5, 7)$
84. Horizontal line, through $(-6, 8)$
85. Through $(6, 0)$, perpendicular to $y = 8$
86. Through $(10, 12)$, perpendicular to $x = -2$

(3.6) Determine which of the following are functions
87. $\{(7, 1), (7, 5), (2, 6)\}$
88. $\{(0, -1), (5, -1), (2, 2)\}$
89. $7x - 6y = 1$
90. $y = 7$
91. $x = 2$
92. $y = x^3$
93. $y = x^2$

Find the domain and the range of each function graphed.
101. $y = \frac{x}{x - 2}$
102. $y = \frac{4}{x}$

103. $y = \frac{2}{x}$
104. $y = \frac{3}{x}$

MIXED REVIEW

Complete the table of values for each given equation.
105. $2x - 5y = 9$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
</tr>
</tbody>
</table>

106. $x = -3y$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Find the intercepts for each equation.
107. $2x - 3y = 6$
108. $-5x + y = 10$

Graph each linear equation.
109. $x - 5y = 10$
110. $x + y = 4$
111. $y = -4x$
112. $2x + 3y = -6$
113. $x = 3$
114. $y = -2$

Find the slope of the line that passes through each pair of points.
115. $(3, -5)$ and $(4, 2)$
116. $(1, 3)$ and $(-6, -8)$

Find the slope of each line.
117. $y = 3x - 2$
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Use the line graph to answer Exercises 125 through 128.

**U.S. Beef Production**

Source: U.S. Department of Agriculture, Economic Research Service

125. Which year shows the greatest production of beef? Estimate production for that year.

126. Which year shows the least production of beef? Estimate production for that year.

127. Which years had beef production greater than 25 billion pounds?

128. Which year shows the greatest increase in beef production?

**CHAPTER 3 TEST**

Graph the following.

1. \( y = \frac{1}{2}x \)
2. \( 2x + y = 8 \)
3. \( 5x - 7y = 10 \)
4. \( y = -1 \)
5. \( x - 3 = 0 \)

Find the slopes of the following lines.

6. 

7. 

11. Determine the slope and the \( y \)-intercept of the graph of \( 7x - 3y = 2 \).

12. Determine whether the graphs of \( y = 2x - 6 \) and \( -4x = 2y \) are parallel lines, perpendicular lines, or neither.

Find equations of the following lines. Write the equation in standard form.

13. With slope of \( -\frac{1}{4} \) through (2, 2)
14. Through the origin and (6, -7)
15. Through (2, -5) and (1, 3)

16. Through (-5, -1) and parallel to \( x = 7 \)
17. With slope \( \frac{1}{8} \) and \( y \)-intercept (0, 12)

Which of the following are functions?

18. 

19.
Given the following functions, find the indicated function values.
20. \( h(x) = x^3 - x \)
   a. \( h(-1) \)   b. \( h(0) \)   c. \( h(4) \)

21. Find the domain of \( y = \frac{1}{x + 1} \).

Find the domain and the range of each function graphed.
22.  

23.  

24. If \( f(7) = 20 \), write the corresponding ordered pair.

Use the bar graph below to answer Exercises 25 and 26.

### Average Water Use Per Person Per Day for Selected Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Liters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uganda</td>
<td>40</td>
</tr>
<tr>
<td>Mexico</td>
<td>150</td>
</tr>
<tr>
<td>Denmark</td>
<td>250</td>
</tr>
<tr>
<td>China</td>
<td>300</td>
</tr>
<tr>
<td>Italy</td>
<td>350</td>
</tr>
<tr>
<td>Australia</td>
<td>400</td>
</tr>
</tbody>
</table>

25. Estimate the average water use per person per day in Denmark.
26. Estimate the average water use per person per day in Australia.

Use this graph to answer Exercises 27 through 29.

**Average Monthly High Temperature:**
Portland, Oregon

<table>
<thead>
<tr>
<th>Month (1 = January)</th>
<th>Temperature (degrees Fahrenheit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: The Weather Channel Enterprises, Inc.

27. During what month is the average high temperature the greatest?
28. Approximate the average high temperature for the month of April.
29. During what month(s) is the average high temperature below 60°F?

**CHAPTER 3 CUMULATIVE REVIEW**

1. **Insert \(<\), \(\geq\), or \(=\) in the space between each pair of numbers to make each statement true.**
   a. 2 3
   b. 7 4
   c. 72 27

2. **Write the fraction \( \frac{56}{64} \) in lowest terms.**

3. **Multiply \( \frac{2}{15} \) and \( \frac{5}{13} \). Write the product in lowest terms.**

4. **Add: \( \frac{10}{3} + \frac{5}{21} \)**

5. **Simplify: \( \frac{3 + |4 - 3| + 2^2}{6 - 3} \)**

6. **Simplify: \( 16 \cdot 3 \cdot 3 + 2^4 \)**

7. **Add.**
   a. \( -8 + (-11) \)
   b. \( -5 + 35 \)
   c. \( 0.6 + (-1.1) \)
   d. \( -\frac{7}{10} + \left( -\frac{1}{10} \right) \)
   e. \( 11.4 + (-4.7) \)
   f. \( -\frac{3}{8} + \frac{2}{5} \)

8. **Simplify: \(|9 + (-20)| + | -10 |\)**

9. **Simplify each expression.**
   a. \( -14 - 8 + 10 - (-6) \)
   b. \( 1.6 - (-10.3) + (-5.6) \)
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10. Simplify: $-9 - (3 - 8)$

11. If $x = -2$ and $y = -4$, evaluate each expression.
   a. $5x - y$
   b. $x^4 - y^2$
   c. $\frac{3x}{2y}$

12. Is $-20$ a solution of $\frac{x}{-10} = 2$?

13. Simplify each expression.
   a. $10 + (x + 12)$
   b. $-3(7x)$

14. Simplify: $(12 + x) - (4x - 7)$

15. Identify the numerical coefficient in each term.
   a. $-3y$
   b. $22x^4$
   c. $y$
   d. $-x$
   e. $\frac{x}{7}$

16. Multiply: $-5(x - 7)$

17. Solve $x - 7 = 10$ for $x$.

18. Solve: $5(3 + x) - (8x + 9) = -4$

19. Solve: $12a - 8a = 10 + 2a - 13 - 7$

20. Solve: $\frac{x}{4} - 1 = -7$

21. If $x$ is the first of three consecutive integers, express the sum of the three integers in terms of $x$. Simplify if possible.

22. Solve: $\frac{x}{3} - 2 = \frac{x}{3}$

23. Solve: $\frac{2(a + 3)}{3} = 6a + 2$

24. Solve: $x + 2y = 6$ for $y$.

25. In a recent year, the U.S. House of Representatives had a total of 435 Democrats and Republicans. There were 31 more Democratic representatives than Republican representatives. Find the number of representatives from each party. (Source: Office of the Clerk of the U.S. House of Representatives)

26. Solve: $5(x + 4) \geq 4(2x + 3)$. Write the solution set in interval notation.

27. Charles Pecot can afford enough fencing to enclose a rectangular garden with a perimeter of 140 feet. If the width of his garden is to be 30 feet, find the length.

28. Solve $-3 < 4x - 1 < 2$. Write the solution set in interval notation.

29. Solve $y = mx + b$ for $x$.

30. Complete the table for $y = -5x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-10</td>
</tr>
</tbody>
</table>

31. A chemist working on his doctoral degree at Massachusetts Institute of Technology needs 12 liters of a 50% acid solution for a lab experiment. The stockroom has only 40% and 70% solutions. How much of each solution should be mixed together to form 12 liters of a 50% solution?

32. Graph: $y = -3x + 5$

33. Graph $x \leq -1$.

34. Find the $x$- and $y$-intercepts of $2x + 4y = -8$.

35. Solve $-1 \leq 2x - 3 < 5$. Graph the solution set and write it in interval notation.

36. Graph $x = 2$ on a rectangular coordinate system.

37. Determine whether each ordered pair is a solution of the equation $x - 2y = 6$.
   a. $(6, 0)$
   b. $(0, 3)$
   c. $(1, -\frac{5}{2})$

38. Find the slope of the line through $(0, 5)$ and $(-5, 4)$.

39. Determine whether each equation is a linear equation in two variables.
   a. $x + 1.5y = -1.6$
   b. $y = -2x$
   c. $x + y^2 = 9$
   d. $x = 5$

40. Find the slope of $x = -10$.

41. Find the slope of the line $y = -1$.

42. Find the slope and $y$-intercept of the line whose equation is $2x - 5y = 10$.

43. Find an equation of the line with $y$-intercept $(0, -3)$ and slope of $\frac{1}{4}$.

44. Write an equation of the line through $(2, 3)$ and $(0, 0)$. Write the equation in standard form.